Today schedule:

1. Questions & Answers regarding Computational simulation
2. Beam transport and RF acceleration short lecture.(30-40 minutes)
3. 10 minutes break.
4. Demonstration of beam manipulation. Go to ATF control.
5. Questions

Emittance compensation

- After initial acceleration, space-charge field is mainly transverse (beam is long in rest frame).
- Both radial and longitudinal forces scale as γ^{-2}
- Transverse force dependent almost exclusively on local value of current density I/σ^2

$$\sigma_x''(\zeta,s) + \kappa_\beta^2 \cdot \sigma_x(\zeta,s) = \frac{r_e \lambda(\zeta)}{2\gamma^3 \sigma_x(\zeta,s)} + \frac{\varepsilon_{n,x}^2}{2\gamma \sigma_x{}^3(\zeta,s)}$$

$$\zeta = s - v_b t$$
$$I(\zeta) = \lambda(\zeta) \cdot v_b$$

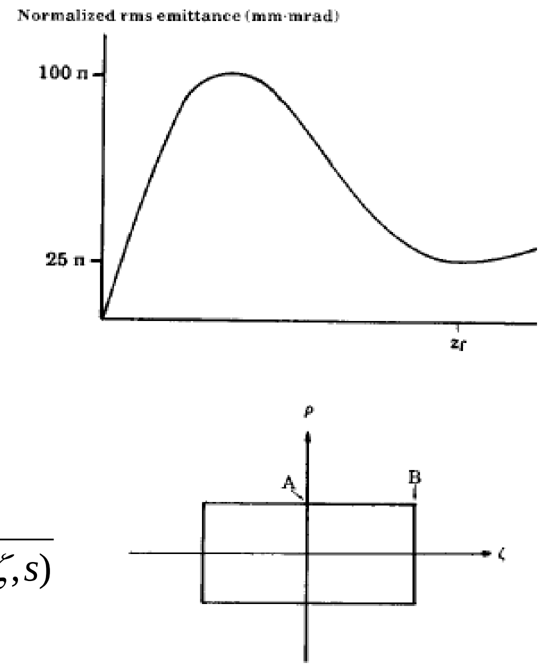


Fig 2 Typical transverse emittance versus beamline p
plot for a photoelectric injector, showing quick initial
and subsequent reduction for a slug beam and physi
scription of a slug beam, with internal coordinates ρ a

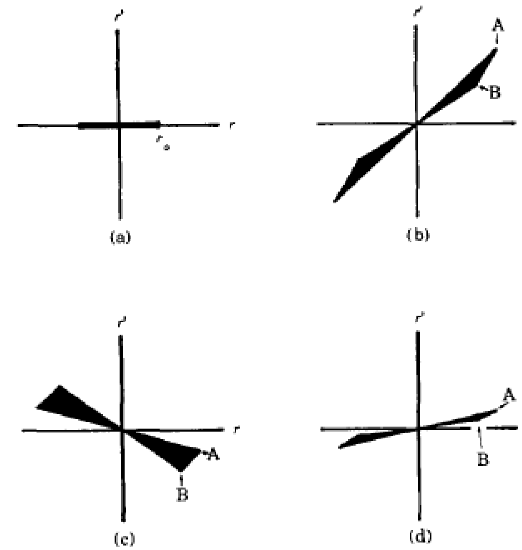


Fig. 3. Transverse phase-space plots showing emittance growth and reduction. (a) Initial phase-space plot with very small emittance. (b) Phase space plot after drift z_1 to lens, showing the emittance growth due to the different expansion rates of points A and B. (c) Phase-space plot immediately after lens, showing rotation due to the lens. The emittance is unchanged because we assume the lens is linear. (d) Phase space plot after drift z behind lens, showing the emittance reduction due to the different expansion rates of points A and B.

Simple model how the emittance compensation works [*]

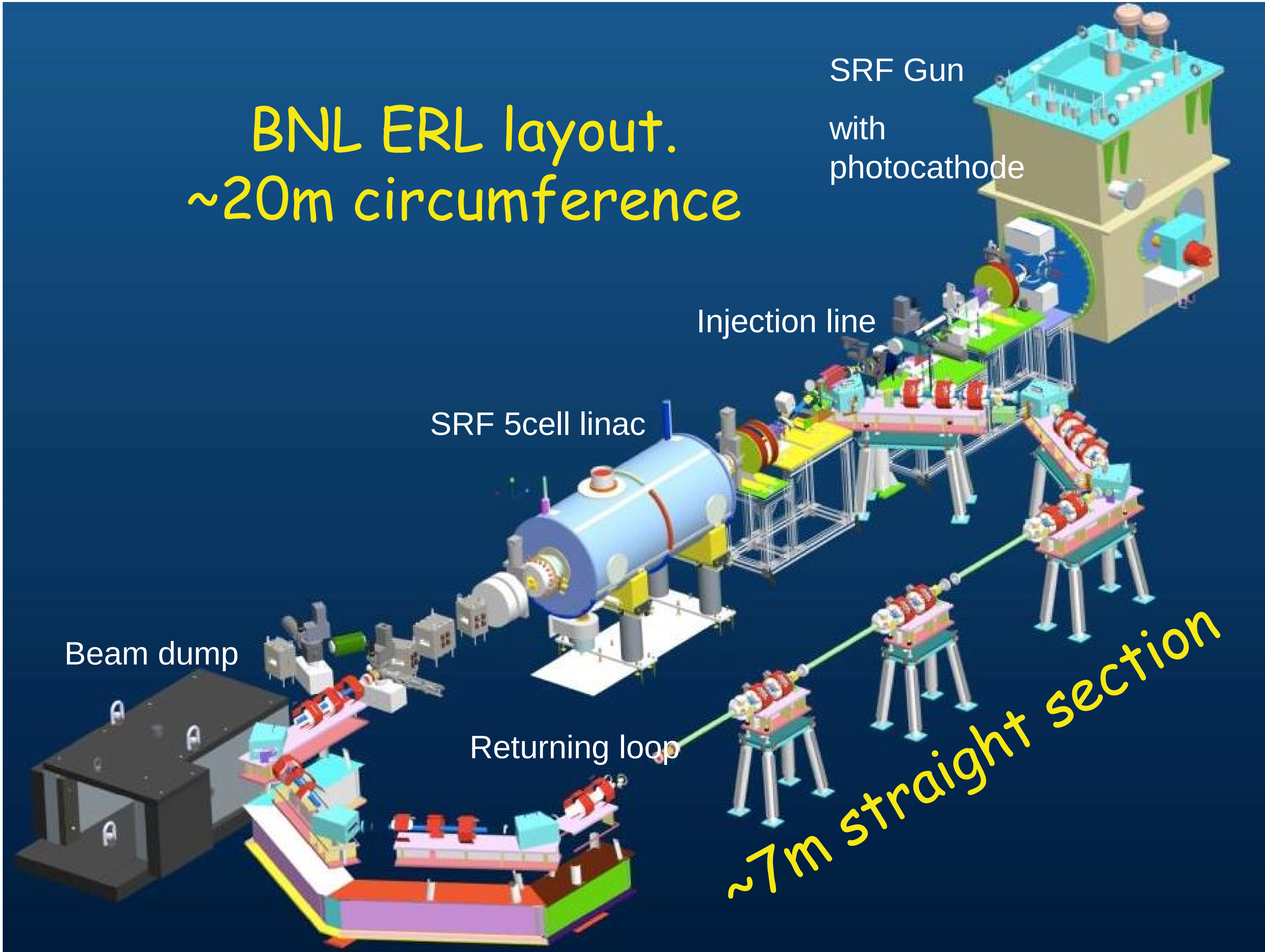
[*] B.E . Carlsten. New photoelectric injector design for the Los Alamos National Laboratory XUV FEL accelerator. NIMA 285 (1989) 313-319

Beam transport and acceleration

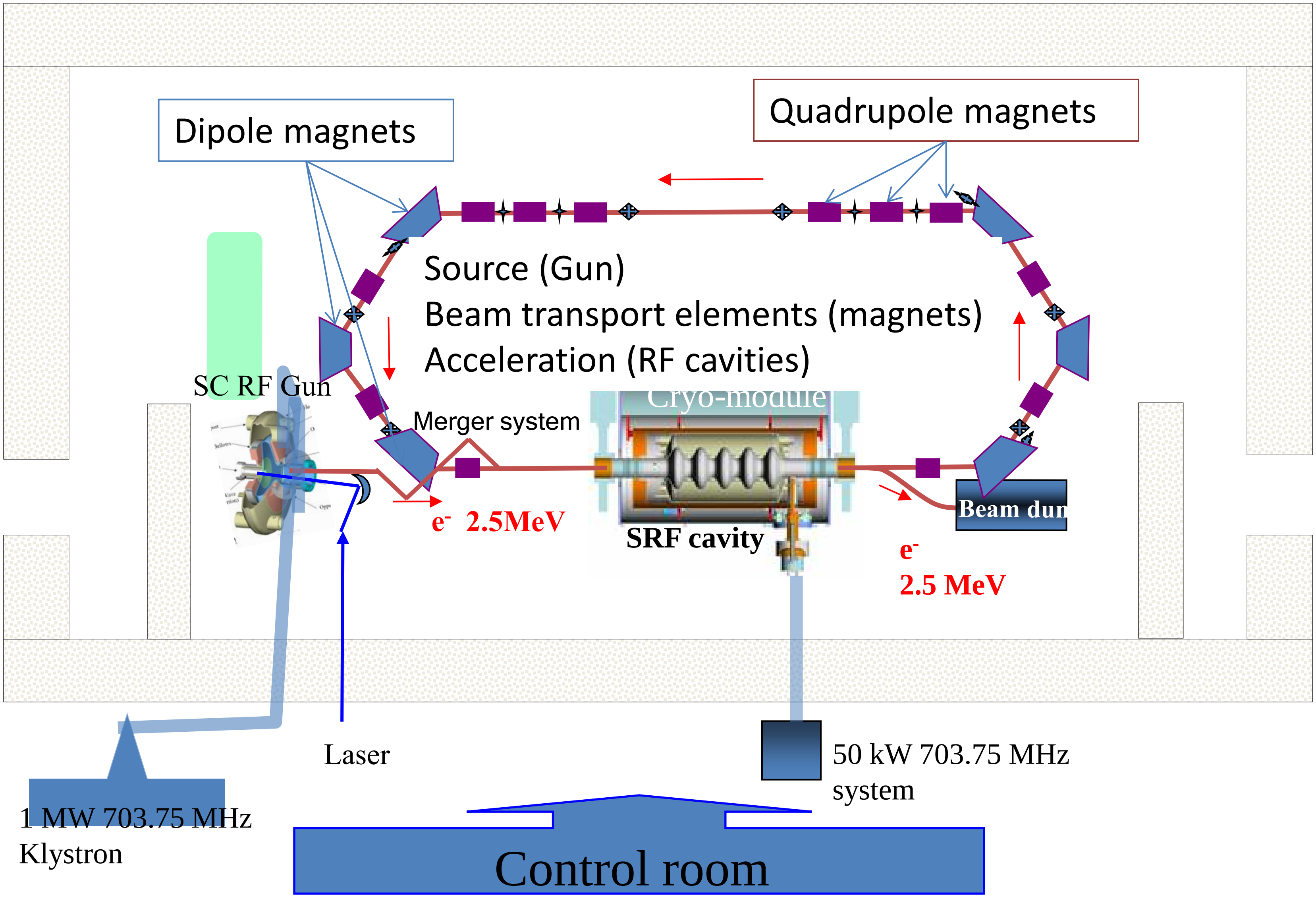
D.Kayran

March 7, 2016

PHY 542 Fundamentals of Accelerator Physics and Technology with Simulations and Measurements Lab


BNL ERL layout.
~20m circumference
SRF Gun
with
photocathode
Injection line
SRF 5cell linac
Beam dump
Returning loop
~7m straight section

Schematic Layout of the BNL ERL



Main accelerators components

- Source
- Beam transport
- Beam Acceleration

- Each particle is defined in 6-D space (coordinates and momentums) $\vec{x} = (x, p_x, y, p_y, z, p_z)$

- In accelerators physics is more convenient to use reference particle and paraxial approximation pz>>px,py then:

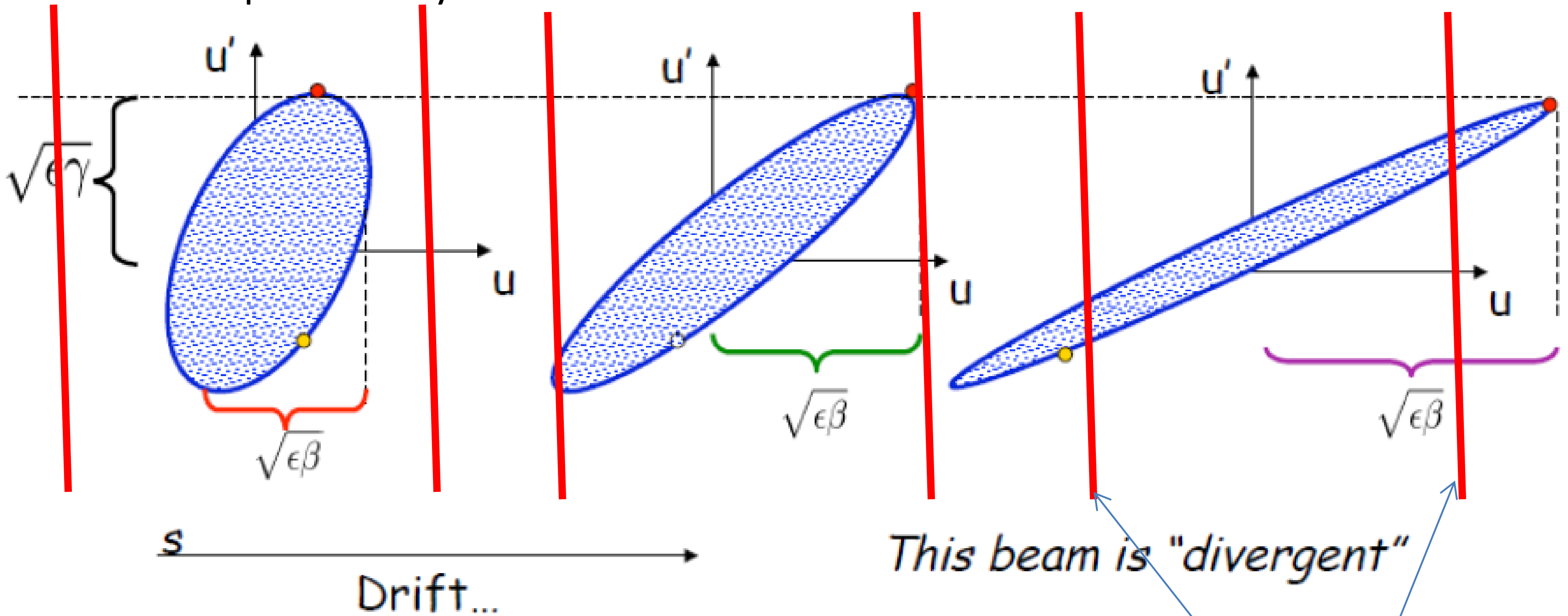
$$\vec{x} = (x, x', y, y', c\Delta t, \Delta E/E)$$

$$x' = \frac{p_x}{p_z} \quad y' = \frac{p_y}{p_z}$$

- $\Delta t, \Delta E$-it s time and energy difference energy from reference particle.

Beam phase space modification drift space only

If there is no coupling between X and Y we can work with 2D phase spaces
For example u=x or y

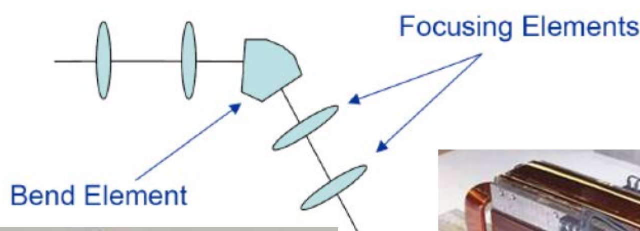


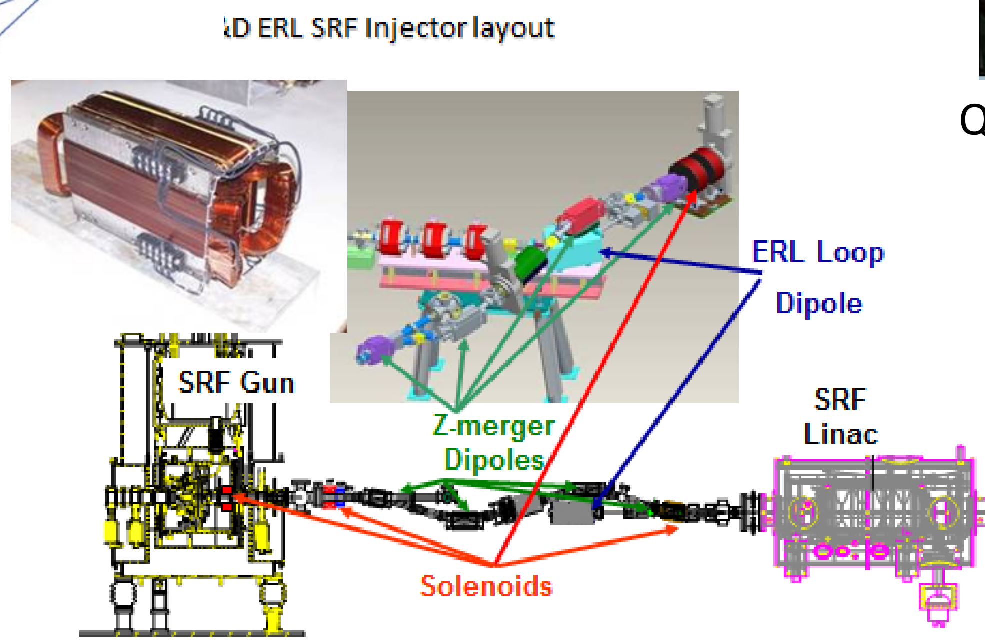
Eventually beam spreads out and hits the aperture
Focusing is needed.

Vacuum pipe aperture
radius=a (+/- a)

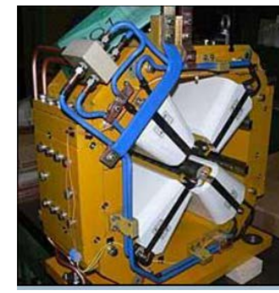
Magnetic lattice

- Usually the set of different kind of magnets is needed in order to successfully propagate charge beam through the system.



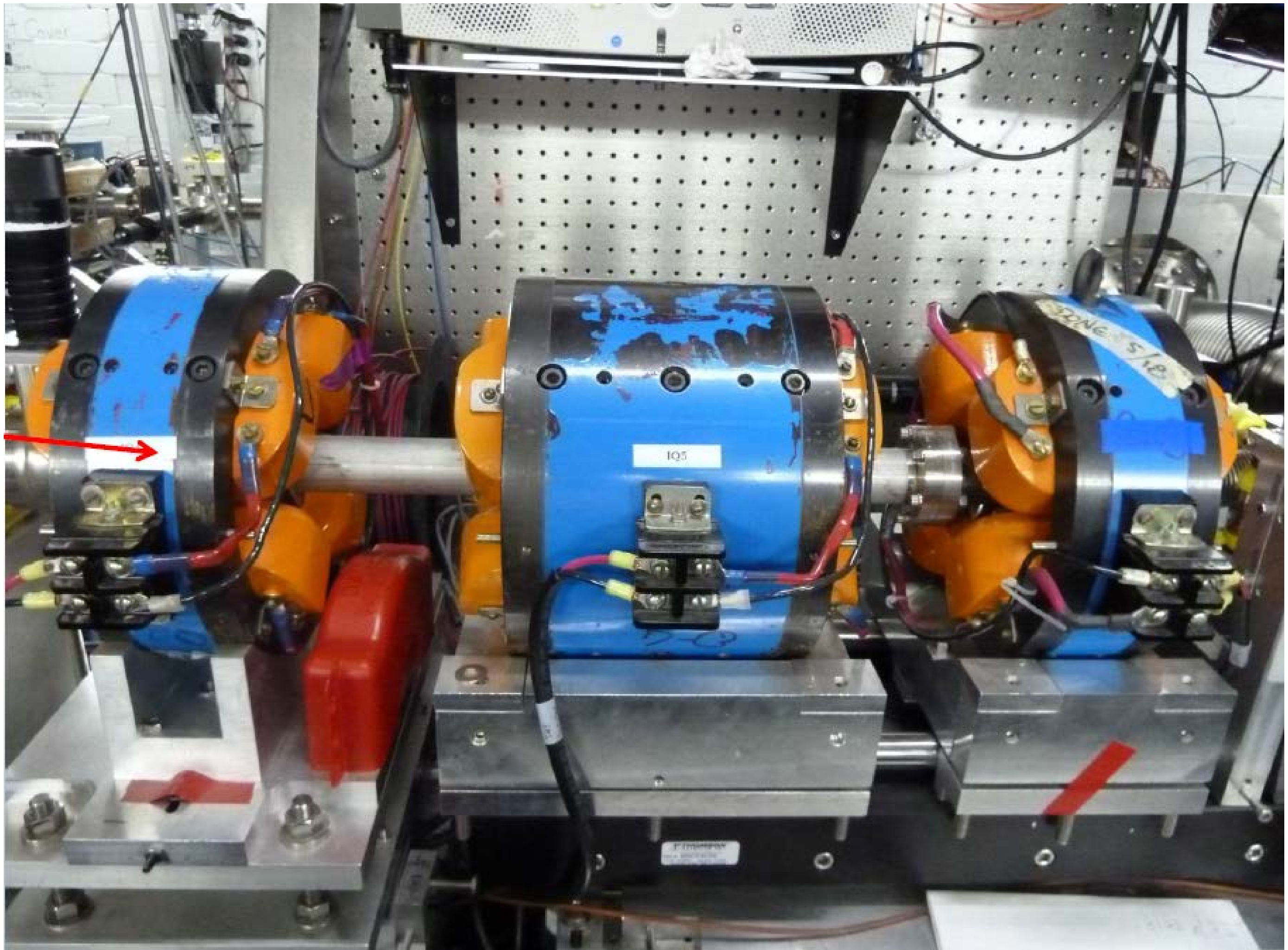


Dipoles



Quadrupoles

ATF qudrupoles



How we can say these are quadrupoles?

Why magnetic field not electric field?

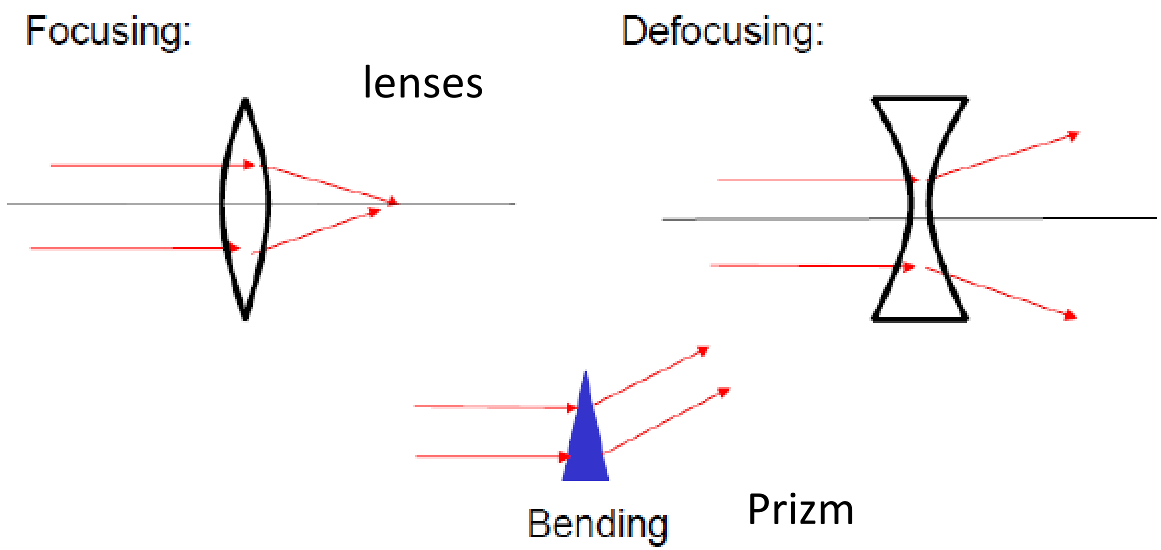
Ratio of magnetic and electric forces

$$F = q(E + v \times B) \Rightarrow \frac{F_M}{F_E} = \frac{vB}{E} \Rightarrow \frac{F_M}{F_E} = 1 \Rightarrow E = vB$$

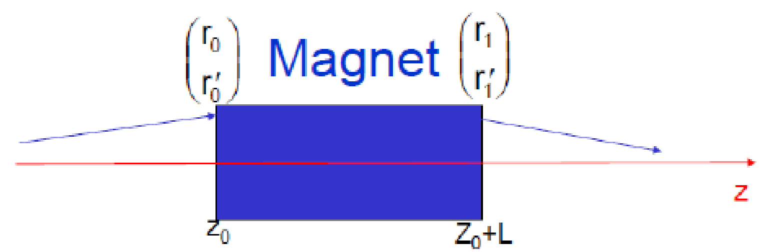
- For ultra-reletivistic particles v~c
  - B=1T is equivalent to E=300MV/m!!!!
- For low energy (v=0.01c)
  - B=1T is equivalent of E=3MV/m

- Electrostatic accelerators existed but the use of such systems are very limited of low energy!!

The light optics similarity



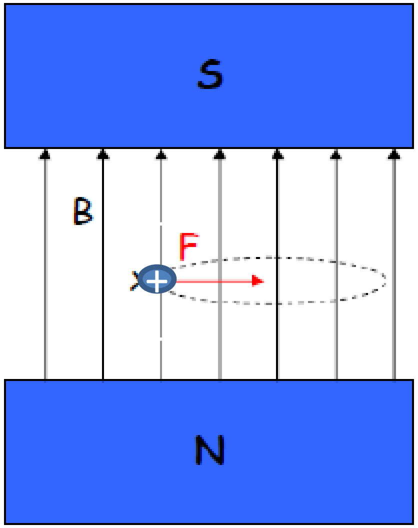
The same matrix formalism can be adopted in first order and linear approximation.
Vectors (x,x') and (y, y')

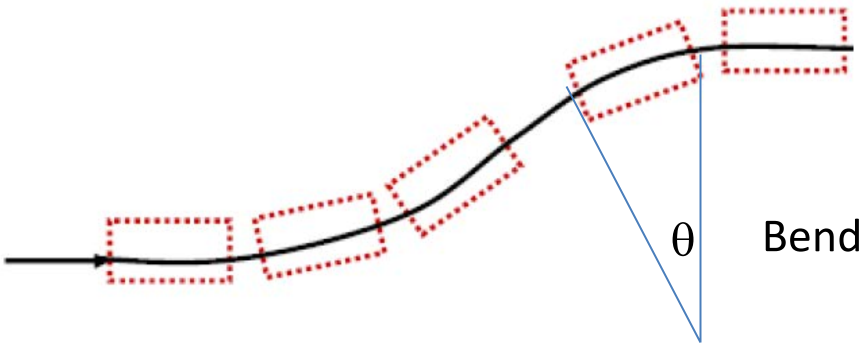


$$\begin{pmatrix} r_1 \\ r_1' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} r_0 \\ r_0' \end{pmatrix} = \mathbf{M} \begin{pmatrix} r_0 \\ r_0' \end{pmatrix}$$

Bending magnets

- A dipole magnet with constant magnetic field
- Positive particle coming in the screen will bend to the right.
- Using combination of the dipoles one could create any kind of transport lines.





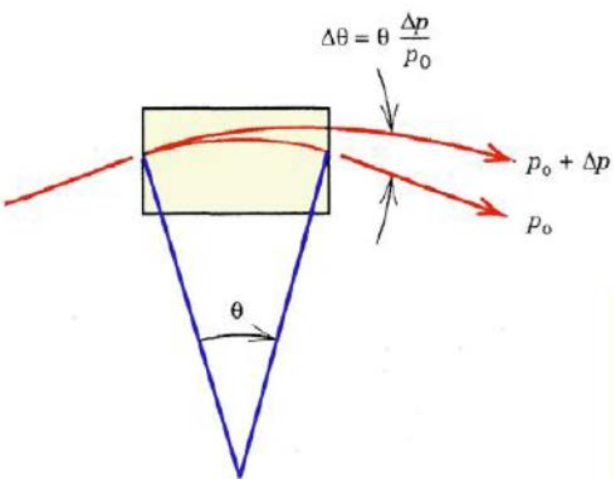
Bending angle

$$\theta = \frac{e}{p_0}\int_{s_1}^{s_2} B\,dl = \frac{e}{B\rho}\int_{s_1}^{s_2} B\,dl$$

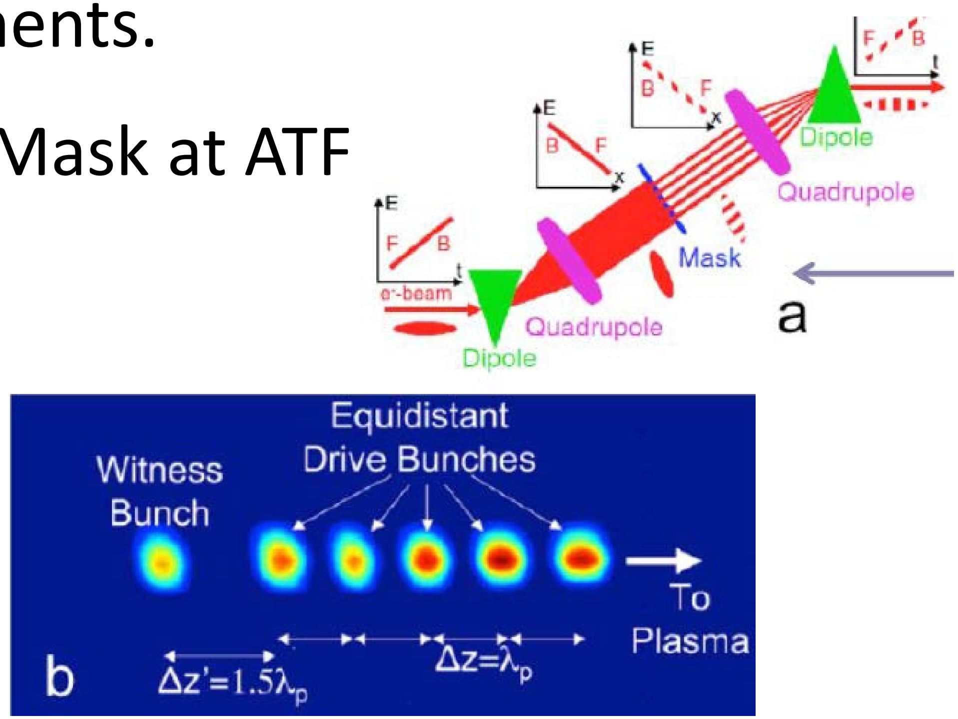
Where, p_0 is the momentum and $B\rho = p_0/e$ is the <u>momentum ‘rigidity’</u> of the beam.

Dispersion

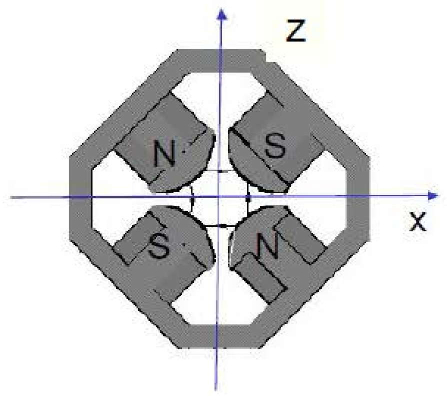
- Particle with different momentum will be bend on different angle
- Can cause beam quality degradation but also used for some experiments.

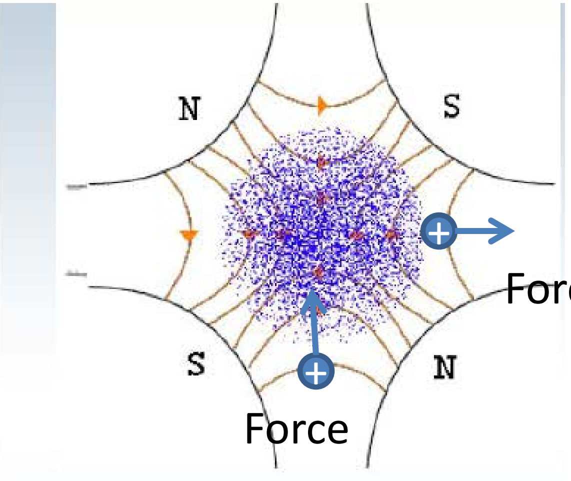


- Mask at ATF



Quadrupoles





$$B = B_1 (z\hat{x} + x\hat{z})$$

- Due to special field symmetry focus beam in one direction but defocus in other.
- Particles moving at axis are not experience any force.

Quadrupoles(cont.)

- Particle displaced by (x,z) from the ceneter

$$\boxed{\mathrm{B} = \mathrm{B}_1\left(\mathrm{z}\hat{\mathrm{x}} + \mathrm{x}\hat{\mathrm{z}}\right)}$$

$$\vec{\mathrm{F}} = \mathrm{evB}_1\hat{\mathrm{s}} \times \left(\mathrm{z}\hat{\mathrm{x}} + \mathrm{x}\hat{\mathrm{z}}\right) = -\mathrm{evB}_1\mathrm{z}\hat{\mathrm{z}} + \mathrm{evB}_1\mathrm{x}\hat{\mathrm{x}}$$

the equations of motion become:

$$\frac{1}{\mathrm{v}^2}\frac{\mathrm{d}^2\mathrm{x}}{\mathrm{dt}^2} = \frac{\mathrm{eB}_1}{\gamma\mathrm{mv}}\mathrm{x}, \quad \frac{1}{\mathrm{v}^2}\frac{\mathrm{d}^2\mathrm{z}}{\mathrm{dt}^2} = -\frac{\mathrm{eB}_1}{\gamma\mathrm{mv}}\mathrm{z}$$

or $\qquad \dfrac{d^2x}{ds^2} = x'' = \kappa x \quad \dfrac{d^2z}{ds^2} = -\kappa z \qquad$ where $\qquad \kappa = \dfrac{eB_1}{\gamma mv}$

When matrix transformation from entrance to exit of quadrupole :

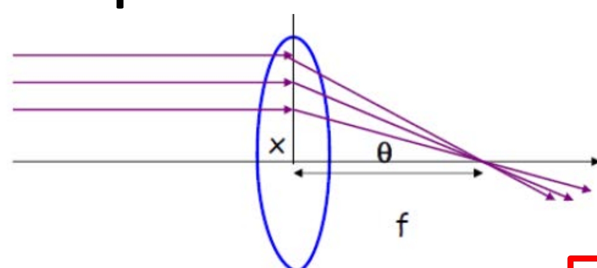
$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos\sqrt{\kappa}L & \frac{1}{\sqrt{\kappa}}\sin\sqrt{\kappa}L \\ -\sqrt{\kappa}\sin\sqrt{\kappa}L & \cos\sqrt{\kappa}L \end{pmatrix}\begin{pmatrix} x_0 \\ x0' \end{pmatrix} \qquad \begin{pmatrix} z \\ z' \end{pmatrix} = \begin{pmatrix} \cosh\sqrt{\kappa}L & \frac{1}{\sqrt{\kappa}}\sinh\sqrt{\kappa}L \\ \sqrt{\kappa}\sinh\sqrt{\kappa}L & \cosh\sqrt{\kappa}L \end{pmatrix}\begin{pmatrix} z_0 \\ z0' \end{pmatrix}$$

Thin lens approximation

- For thin lens when K<<1/L^2

$$\begin{pmatrix} \cos(\sqrt{K}L) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}L) \\ -\sqrt{K}\sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ -KL & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix}$$

- If the quadrupole is thin enough, the particles coordinate doesn’t change while momentum change. The quad works almost as a optical lens…



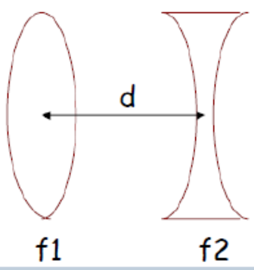
$$\Delta x' = \frac{x}{f}$$

- With only one difference:

Focus in one plane and defocus in other plane

Focus the beam in both directions.

- Using doublets
- Using optical analogy one can calculate



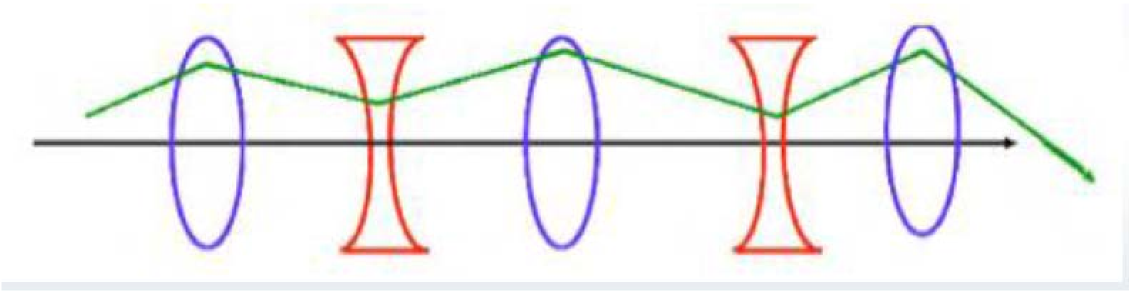
The combined f is:

$$\frac{1}{f_{combined}} = \frac{1}{f1} + \frac{1}{f2} - \frac{d}{f1 f2}$$

What if f1 = -f2?

$$f_{combined} = \frac{f1^2}{d}$$

- A quadrupole doublet is focusing in both planes.
- Strong focusing by sets of quadrupole doublets with alternative gradient. Could keep beam inside vacuum chamber.

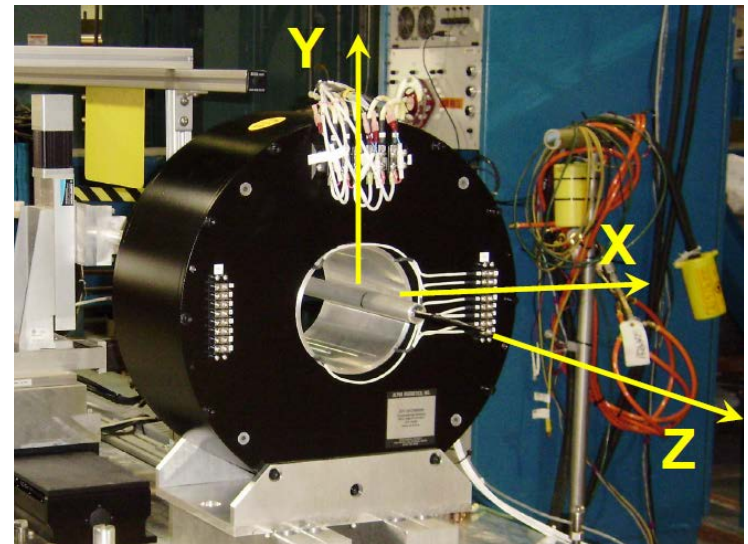


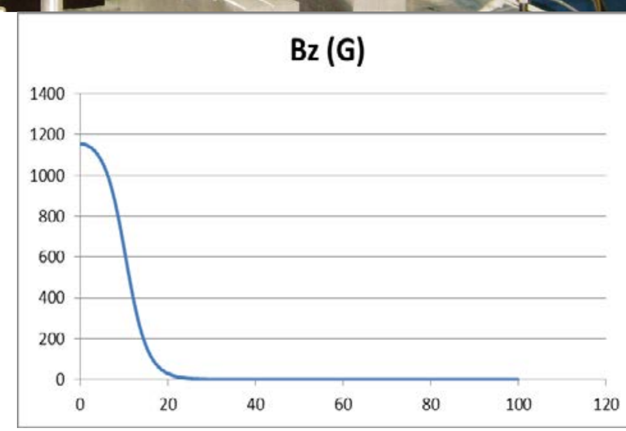
Solenoid

- A solenoid is a set of helical coils.
- Typically solenoid radius is smaller then the length.
- Magnetic field is generated along the axis line.
- Solenoid couples X and Y motion.
- Solenoid produced focusing in both direction

$$1/f = e\int Bz^2 dz/(2pc)^2$$

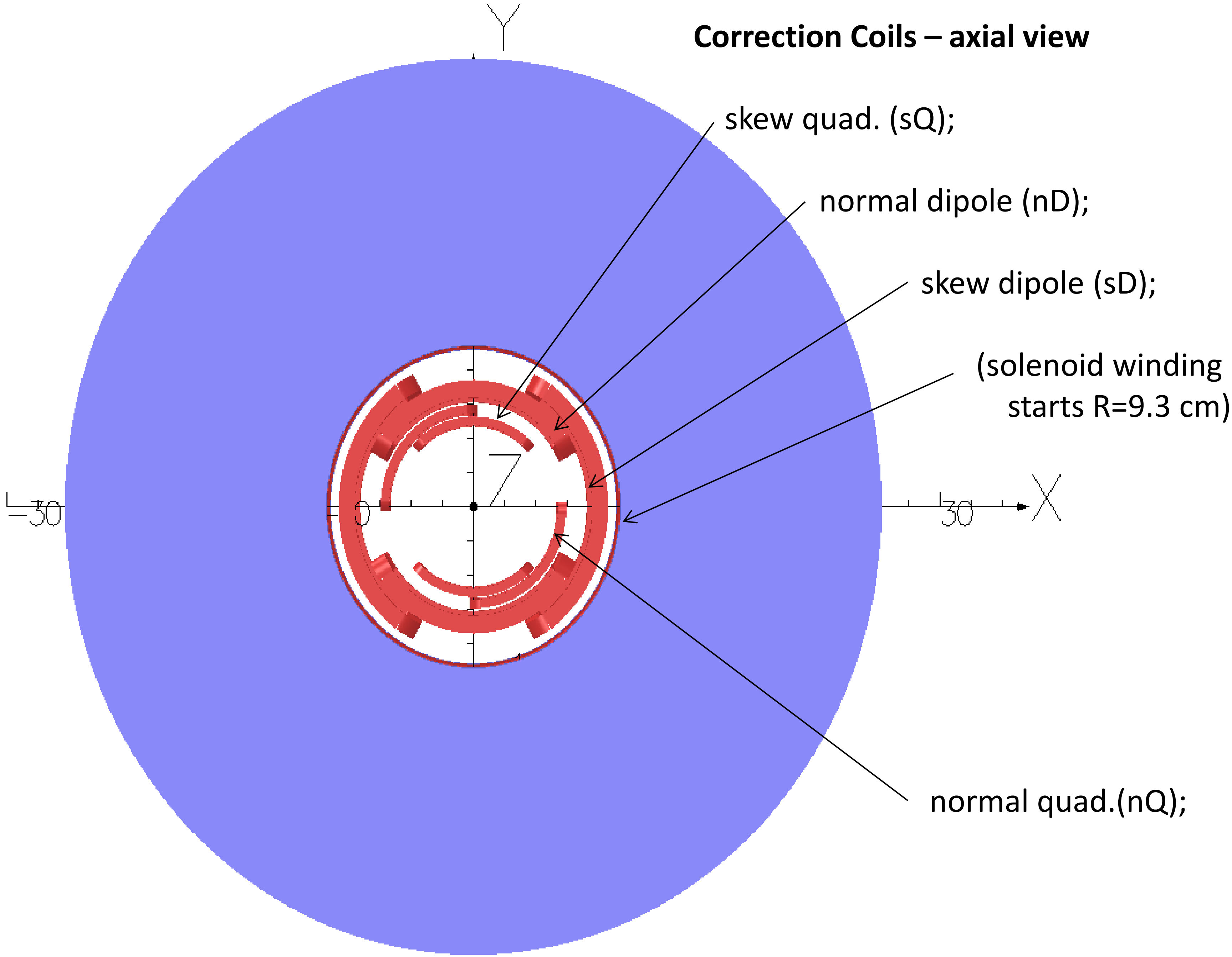
- Solenoids are preferable at low energy



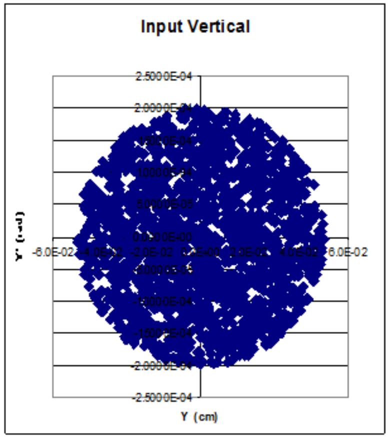


Back up

Correction Coils – axial view



Linear transformation preserves emittance.



ε_{ny}= 0.96 um

β_y= 249 cm

α_y= 0.0012

σ_y=0.25mm

Dipole magnet:

$x=x_0\cos(phi)+x_0'R*\sin(phi)$

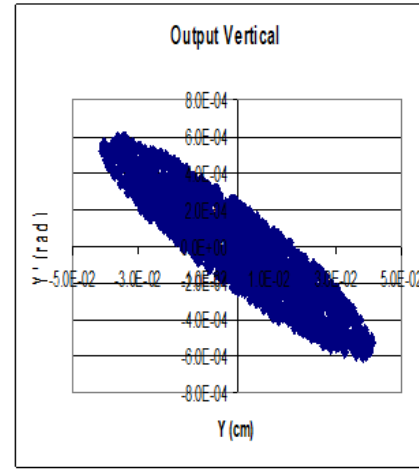
$x'=-x_0/R*\sin(phi)+x_0'\cos(phi)$

Thin focusing lens with f- focal length:

$x=x_0$; $x'=x'_0- x_0/f$

Drift space L - length :

$x=x_0+L*x_0$; $x'=x'_0$



ε_{ny}= 0.960 um

β_y= 168 cm

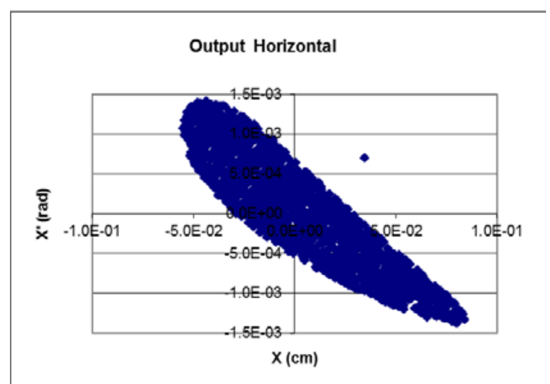
α_y= -2.20

σ_y=0.20 mm

Nonlinear transformation increases emittance:

Thin Sextupole:

$x=x_0$; $x'=x_0'-S*x_0^2$



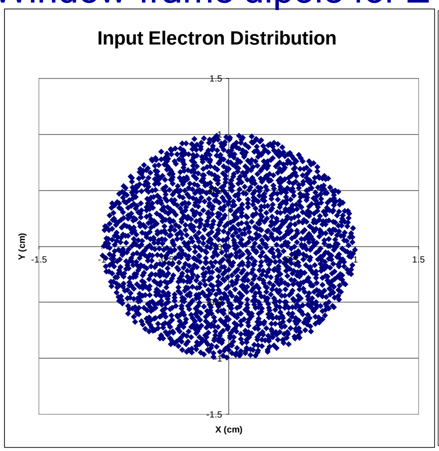
Injection combine function magnets

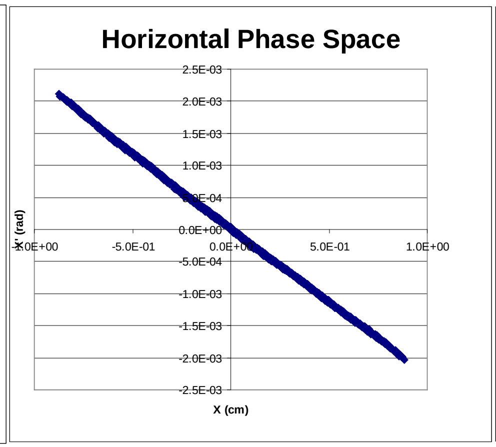
Window-frame dipole for Z-bend

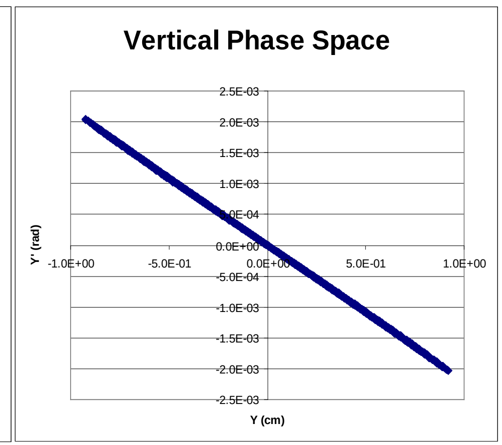
Due to very small real estate and large beam size: each magnet includes 4 sets of coils: 1) vertical bend, 2) quadrupole focusing, 3) sextupole correction and 4) horizontal steering.

The quadrupole coil is used to split focusing equally between the planes

The amount of the sextupole component is controlled by the gap between the yoke and the main dipole coil. A small additional coil in the corners is a sextupole trim coil, intended for use if sextupole component needs to be adjusted to reduce emittance growth







initial emittances 0

After tracking emittances:

ε_{xn}=0.6 mm-mrad

ε_{yn}=0.6 mm-mrad

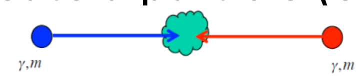
Analysis predicts that the influence of various field components on the emittance growth are complicated by the fact that the beam trajectory bends significantly in the fringe fields. Hence, direct tracking in the calculated fields extracted from Opera3d was used of test beam to evaluate and to minimize influence of magnetic field on the beam emittance

Main accelerators components

- Source
- Beam transport
- Beam Acceleration

Acceleration is needed!!

- In colliders: The minimum energy required to create a particle (or group of particle) with total mass M is: $E_{\min} = Mc^2$



  - High energy colliding particles=>high energy center mass=> massive particles production (cross section σ)
  - luminosity:

$$L = f_c \frac{N_1 N_2}{A} \cong f_c \frac{N_1 N_2}{2\pi \sqrt{\beta_{x1}\varepsilon_{x1} + \beta_{x2}\varepsilon_{x2}} \sqrt{\beta_{y1}\varepsilon_{y1} + \beta_{y2}\varepsilon_{y2}}}$$

Numbers of events

$$N_{A\to B} = \sigma_{A\to B} \cdot L$$

$$\varepsilon_{n,s} = \beta\gamma \sqrt{\langle s^2\rangle\langle s'^2\rangle - \langle ss'\rangle^2}$$

- Normalized emittance ~preserved during acceleration, geometrical emittance reduced ~1/γ.

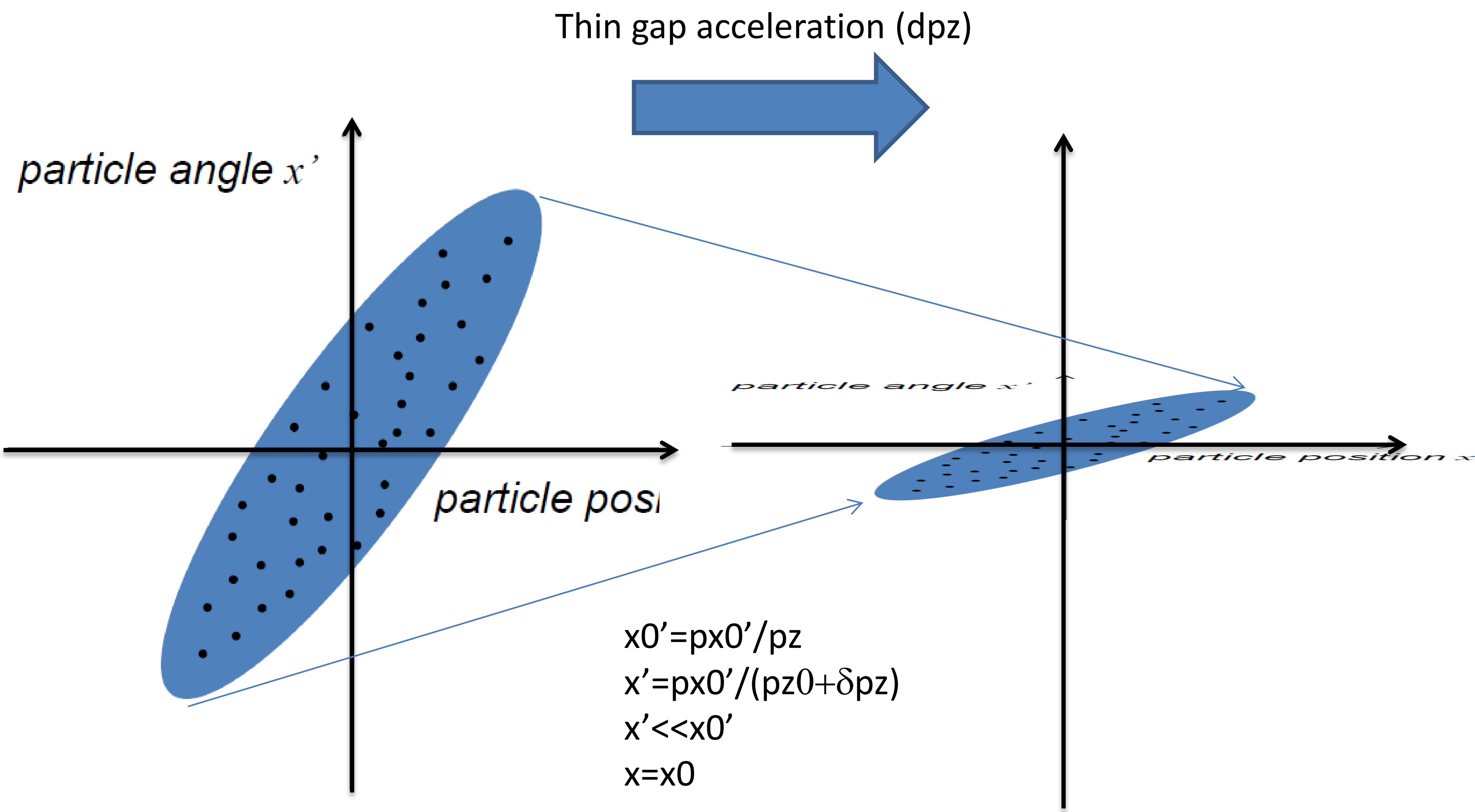
where s is either x or y.

The peak normalized rms brightness is given by

$$B_n = \frac{2I}{\varepsilon_{n,x}\varepsilon_{n,y}}$$

- In light source: Brightness B ~1/γ^2.

Geometrical emittance transformation



x0'=px0'/pz

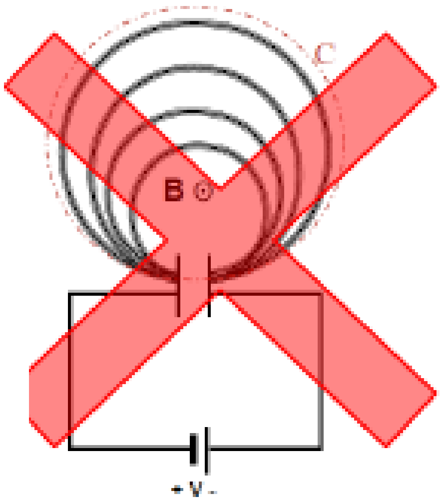
x'=px0'/(pz0+δpz)

x'<<x0'

x=x0

Acceleration

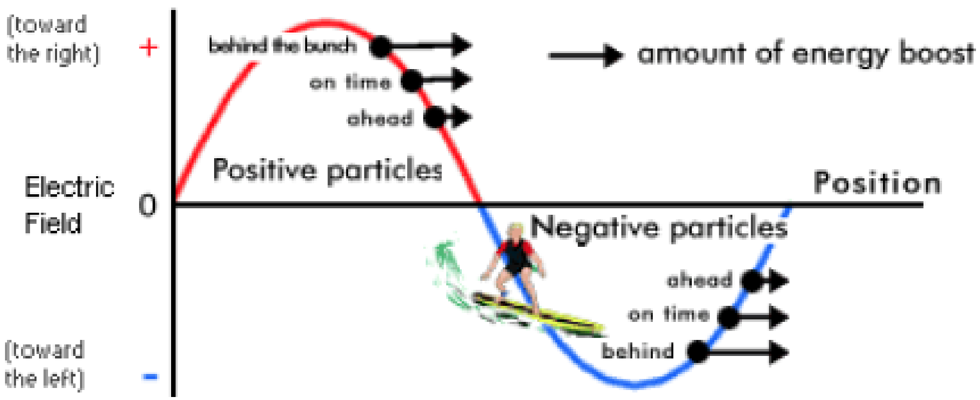
$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right); \quad \frac{dE}{dt} = q\left(\vec{E} \cdot \vec{v}\right);$$



- Single pass acceleration
- Limited by maximum voltage per until discharge. ~1.5 MV in air

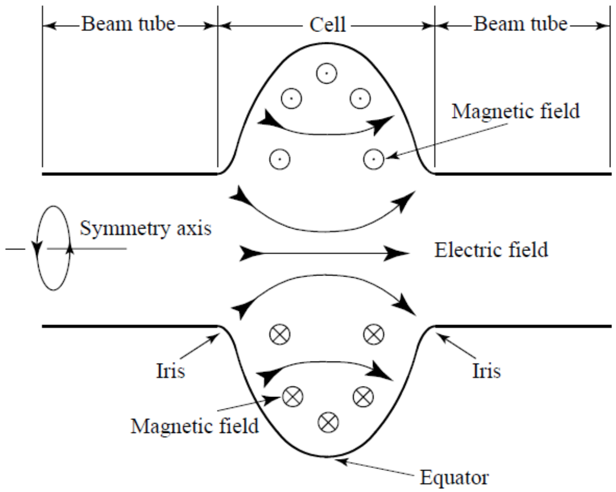
$$\Delta E = e\oint \vec{E} \cdot d\vec{l} = -\frac{e}{c}\frac{\partial}{\partial t}\left(\int \vec{H} \cdot d\vec{s}\right)$$

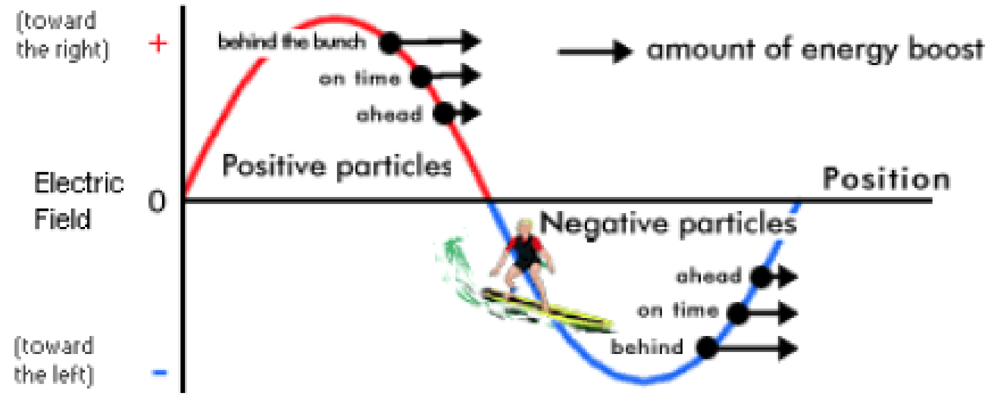
Maxwell equation prohibits multiple acceleration is DC electric field:



- In RF cavities energy gain depends on the phase.
- The main purpose of using RF cavities in accelerators is to add (remove) energy to charged-particle beams at a fast acceleration rate

RF Field acceleration:





The RF field must be synchronous (correct phase relation) with the beam for a sustained energy transfer.

$$E_z(z,t) = E(z)\cos\left(\omega t - \int_0^z k(z)\,dz + \phi\right),$$

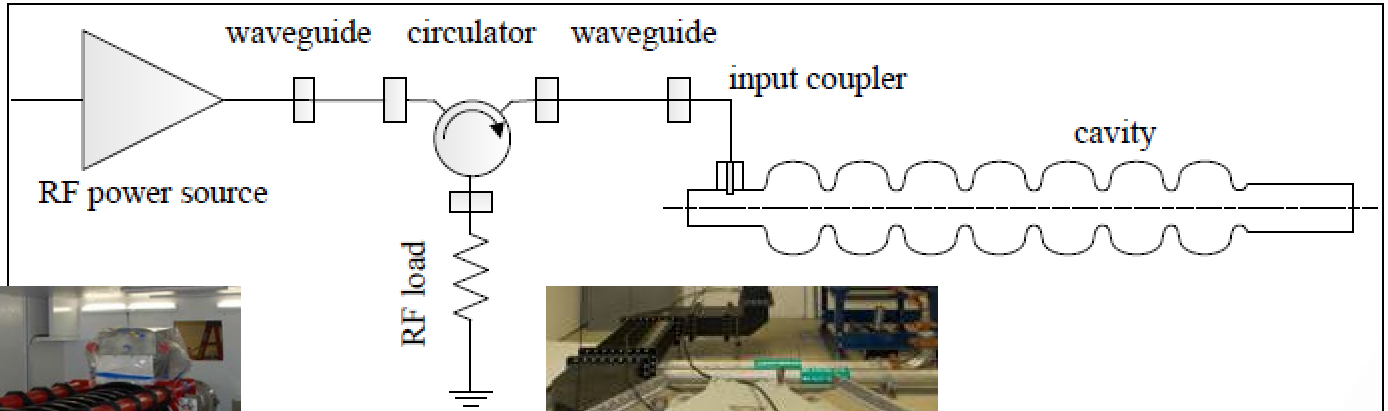
For efficient particle acceleration, the phase velocity of the wave must closely match the beam velocity. If we consider a particle of charge *q* moving along *+z* direction with a velocity at each instant of time equal the phase velocity of the traveling wave, then the electric force on the particle is given by

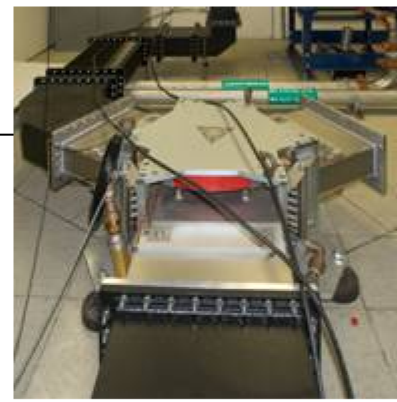
$$F_z = q\,E(z)\,\cos\phi$$

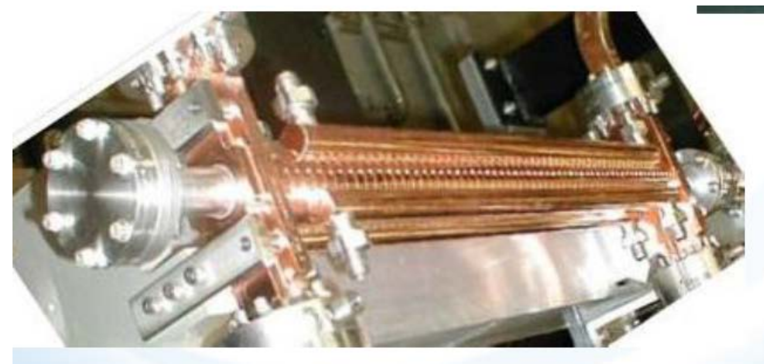
Energy gain

$$\Delta\mathcal{E} = q\int_{-L/2}^{L/2} E(0,z)\cos\left[\omega t(z) + \phi\right]\,dz$$

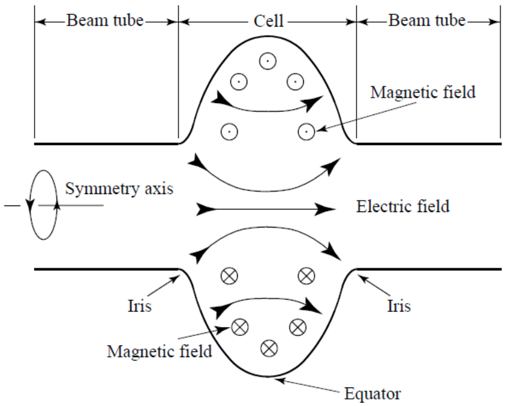
RF Cavity connected to RF power source



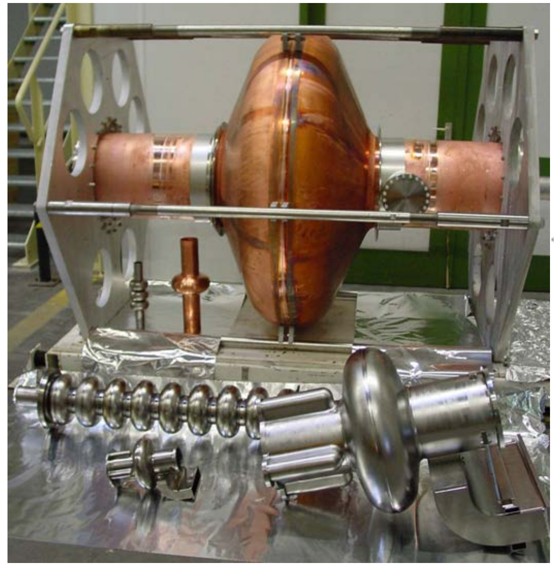


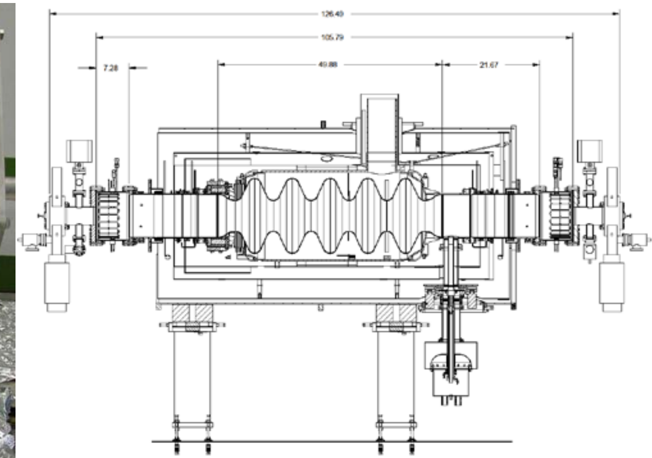


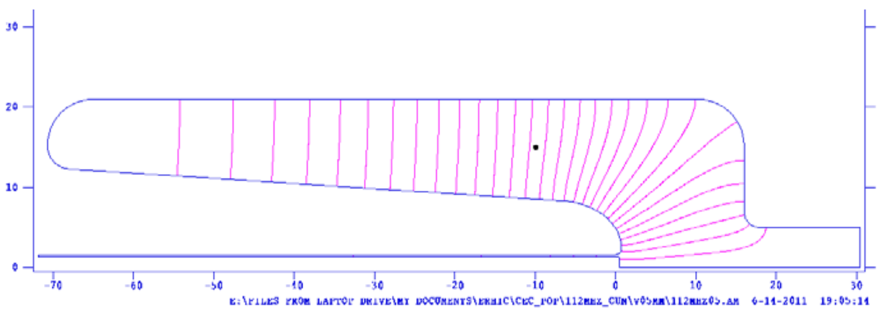
RF cavities



Typical Single cell

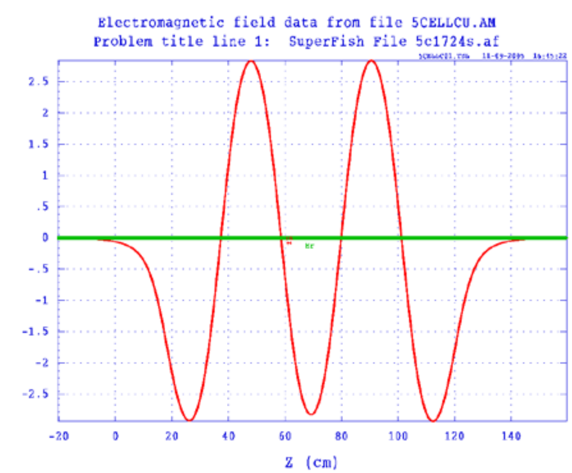




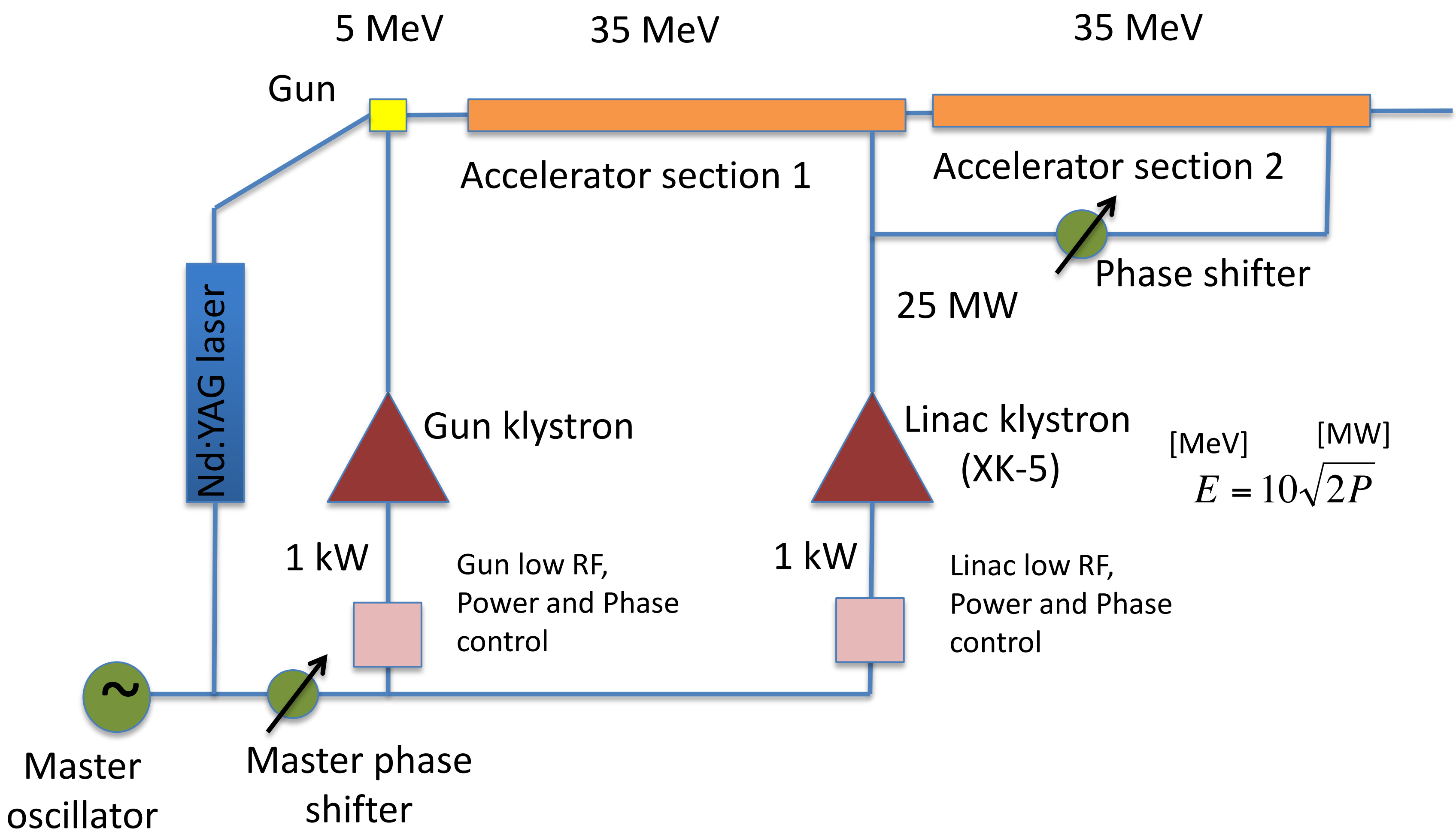


Quarter-wave 112MHz resonator

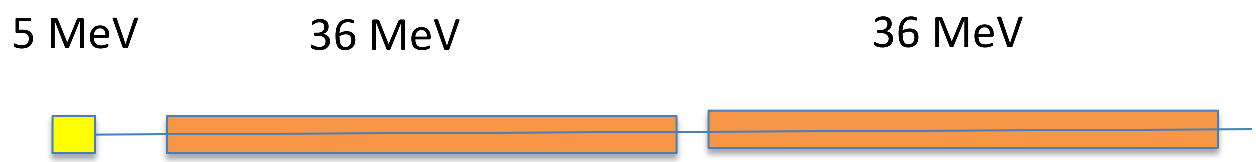
BNL ERL: 5Cell cavity 704MHz



ATF accelerator system



Multi linacs acceleration



$E = E_{inj} + Elinac1 + Elinac2$

$Elinac1 = eU1\cos(phi1)$

$Elinac2 = eU2\cos(phi2)$

If there is enough voltage provided by one linac.
The final energy can be reached by combination different phases.

For ATF:
U1=U2=36MV, Einj=5MeV
Efinal=35MeV

phi1	phi2
65.4	65.4
65.4	-65.4
0.0	99.6
0.0	-99.6
90.0	33.6
-90.0	-33.6
33.6	90.0
-33.6	-90.0

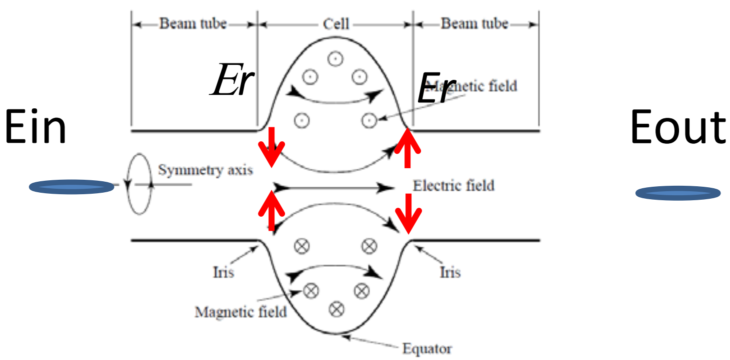
Why one operation could be better then others?

Few things to remember

- Space charge force depends on energy
  - Higher energy => less space charge effects

$$\sigma_x''(\zeta,s)+\kappa_\beta^2\cdot\sigma_x(\zeta,s)=\frac{r_e\lambda(\zeta)}{2\gamma^3\sigma_x(\zeta,s)}+\frac{\varepsilon_{n,x}^2}{2\gamma\sigma_x^{\ 3}(\zeta,s)}$$

- Focusing due to entrance and exit of RF field
  - More energy gain => stronger focusing



$$\Delta p_r = \frac{e}{c}\int E_r dz$$

$$r'_{in} = \frac{\Delta p_{in\_r}}{p_{in\_z}} \qquad r'_{out} = \frac{\Delta p_{out\_r}}{p_{out\_z}}$$

$$\Delta p_{in\_r} \sim -\Delta p_{out\_r}$$

Eout= Ein+ΔE

Entrance kick is larger then exit kick