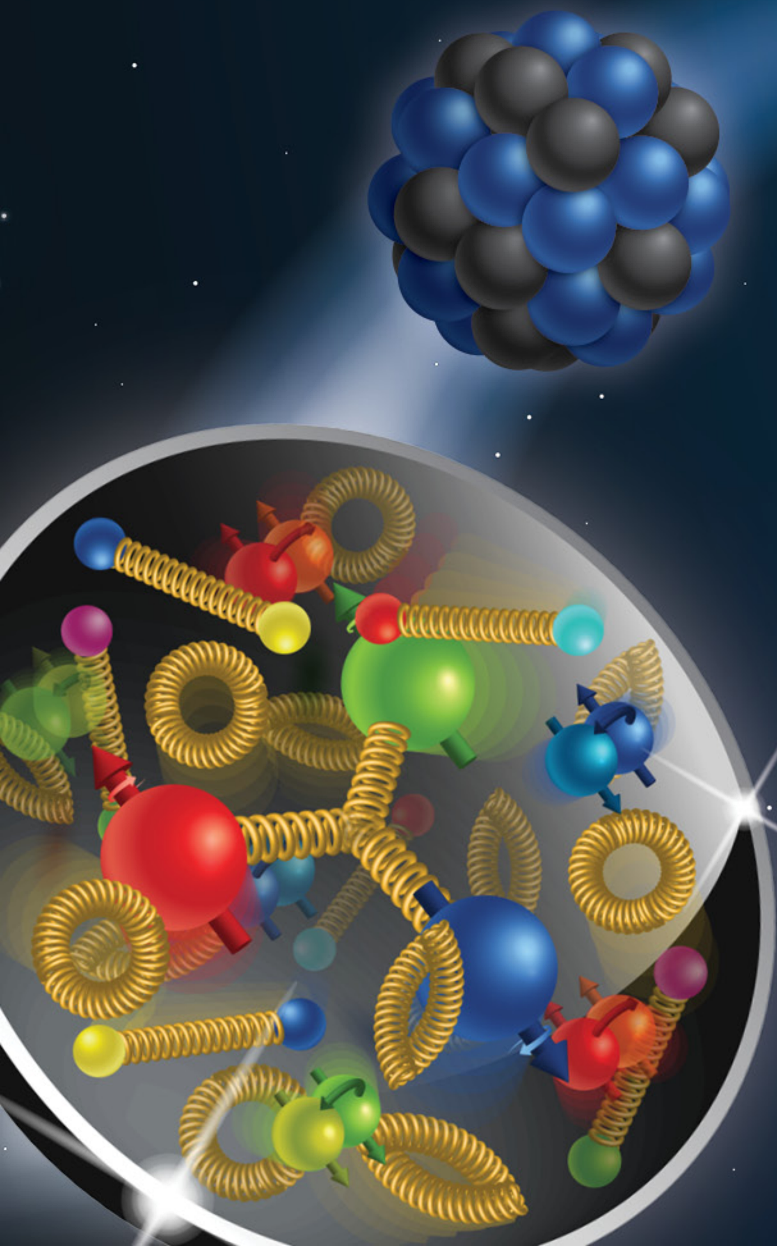

Coherent electron Cooling Part I

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Center for Accelerator Science and Education

CASE seminar August 2 2017

Measure of a Collider Performance is the Luminosity

$$\dot{N}_{events} = \sigma_{A \to B} \cdot L$$

$$L = \frac{f_{coll} \cdot N_1 \cdot N_2}{4\pi\beta^* \varepsilon} \cdot g(\beta^*, h, \theta, \sigma_z)$$

- Why to cool?

Main sources of luminosity limitation in hadron colliders

Large or growing emittance
Hour-glass effect
Crossing angle
Beam Intensity & Instabilities
Beam-Beam effects

Cooling of beam emittances is needed either to increase the luminosity or to mitigate its reduction

Why Coherent electron Cooling?

In many occasions we need to reduce phase volume space occupied by beam., e.g. to cool beam. While electron and positrons have natural strong cooling mechanism via synchrotron radiation with damping time measured in milliseconds, the hadrons at currently available energies do not...
Hence accelerator physicists are always looking for the ways if doing it efficiently. In future eRHIC cooling can boost luminosity by factor of 50... but none of existing cooling techniques can do the job...

Machine		Energy GeV/n	RF Stochastic Cooling, hrs	SR, hrs	e-cooling hrs	CeC
RHIC CeC PoP	*Au*	*26*	-	-	~ 1	*10 sec – local* *30 min - bunch*
eRHIC	p	325	~100	∞	~ 30	~ 0.1hr
LHC	p	7,000	~ 1,000	13/26	∞	~ 1 hr

Main Limitation of Electron Cooling

- Main limitation of electron cooling is its rapidly falling efficiency with the increase of the beam energy $\tau \sim \gamma^{7/2}$ is typical for RHIC's electron cooler design [1,2]
- Cooling protons in LHC at 7 TeV is ~ 10^{10} harder that cooling 9 GeV antiprotons in the Fermilab recycler
- Even cooling protons in RHIC at 250 GeV falls many orders of magnitude bellow requirements for eRHIC

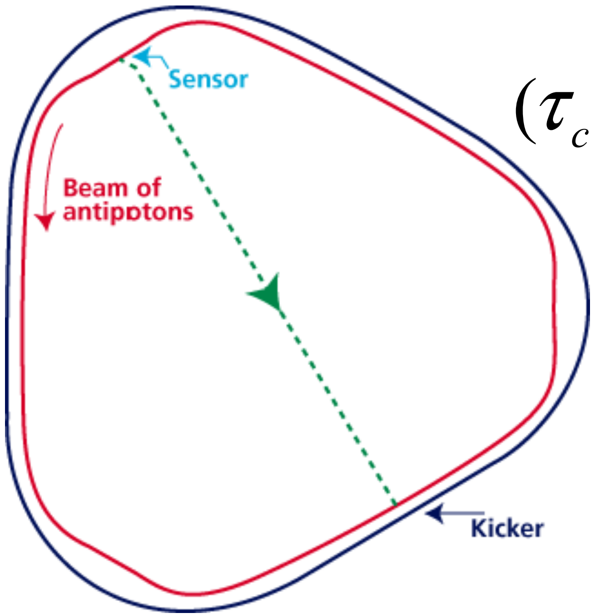
[1] Ilan Ben-Zvi, et. al., NIM **A532**, 177, (2004)
[2] A.V.Fedotov et al, New J. of Physics **8** (2006) 283
[3] S.Nagaitsev et al., Phys. Rev. Lett. 96, 044801 (2006

Stochastic cooling

The van der Meer's demon

The 1984 Nobel prize for accelerator physicist

Imaged by Heritage Auctions, HA.com



$$\langle \tau_c \rangle \geq \frac{N}{2\pi\Delta f}$$

It works!!

S. van der Meer, 1972, Stochastic cooling of betatron oscillations is ISR, CERN/ISR-PO/72-31
S. van der Meer, Rev. Mod.Phys. 57, (1985) p.689

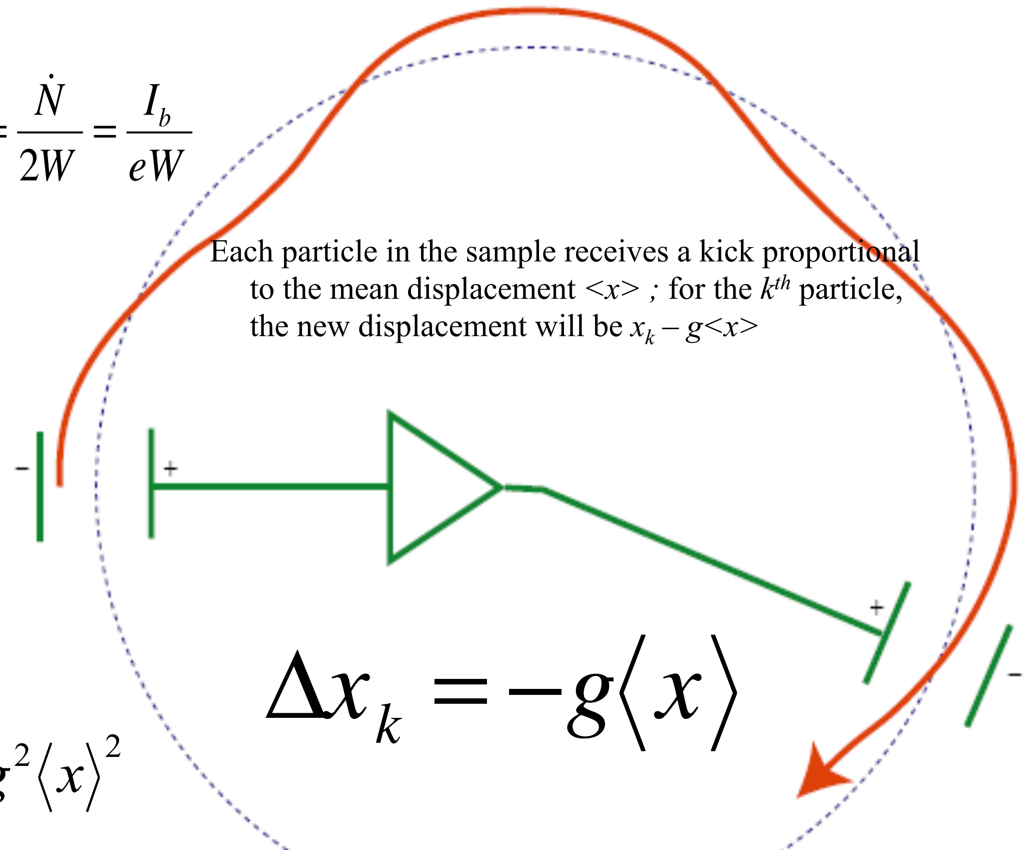
- **Worked well for a coasted low current, large emittance beams**
- **Works beautifully in RHIC for bunched ion beams**

Transverse Stochastic Cooling

For a system with flat response over bandwidth W,

- Sample a particle's motion at one point (with the ***pickup***)
- Correct the angle at another point (using the ***kicker***)

$$N_s = \frac{\dot{N}}{2W} = \frac{I_b}{eW}$$



Each particle in the sample receives a kick proportional to the mean displacement $<x>$; for the k^{th} particle, the new displacement will be $x_k - g<x>$

$$\Delta x_k = -g\langle x \rangle$$

$$\langle x \rangle = \frac{1}{N_s}\sum_i x_i = \frac{1}{N_s} x_k + \frac{1}{N_s}\sum_{i \neq k} x_i$$

$$x_k^2 \to (x_k - g\langle x \rangle)^2 = x_k^2 - 2g x_k \langle x \rangle + g^2 \langle x \rangle^2$$

Stochastic cooling uses ***incoherent*** motion of ***each individual particle***, instead of coherent motion of beam. I.e. it directly relies on finate number of particles in the bunch

Image from: J. Marriner, Nucl.Instrum.Meth. A532 (2004) 11-18

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Transverse Stochastic Cooling

Averaging over all the particles gives

$$\frac{1}{N_s}\sum_k (x_k - g\langle x\rangle)^2 = \frac{1}{N_s}\sum_k x_k^2 - \left(\frac{2g}{N_s} - \frac{g^2}{N_s^2}\right)\sum_k x_k^2$$

Sum of contributions of each particle on itself

$$-\left(\frac{2g}{N_s} - \frac{2g^2}{N_s^2}\right)\sum_{k,i\neq k} x_i x_k \to 0$$

Vanishes since different particle's displacements are uncorrelated!

$$+\frac{g^2}{N_s}\sum_k\left(\frac{1}{N_s}\sum_{i\neq k} x_i\right)^2$$

…which is partially offset by contributions from other particle in sample

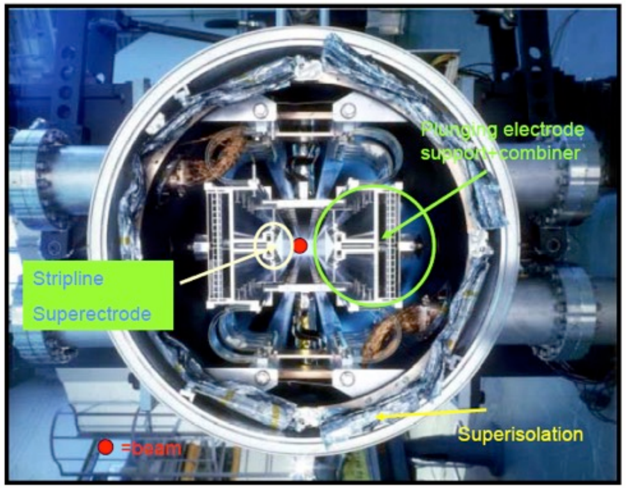
$$= \frac{1}{N_s}\bullet N_s\langle x^2\rangle = \langle x^2\rangle$$

$$\frac{d\langle x^2\rangle}{dn} = -\frac{2g}{N_s}\langle x^2\rangle + \frac{g^2}{N_s}\langle x^2\rangle \qquad \frac{1}{\varepsilon}\frac{d\varepsilon}{dn} = -\left(\frac{2g-g^2}{N_s}\right)$$

RF SC limitations

- In ideal world with unlimited power of HF BB amplifiers there is no energy dependence of efficiency of stochastic cooling
- Main limitation of stochastic cooling (for a fixed bandwidth) is that its cooling time directly proportional to linear density of the particles and modern proton beams with ~ 10^{11} p/nsec are simply to dense

$$\frac{\tau}{T_{rev}} \sim \frac{\dot{N}}{W}$$

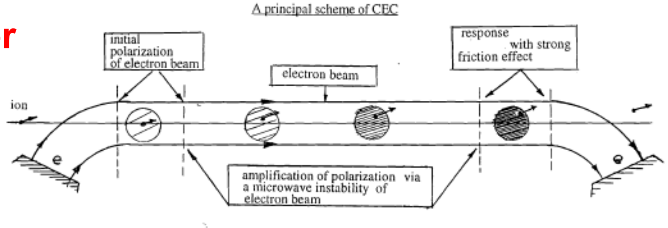


- **Thus, SC with W ~ 1 GHz can cool protons with ~ 10^{11} p/nsec in LHC (T ~10^{-4} sec) in about 10^7 sec, i.e. in 3 months (in RHIC it would take ~ 10 days)**

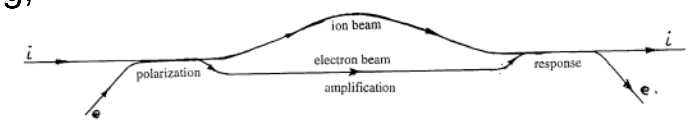
Yaroslav Derbenev Started Discussing Possibility of Coherent electron Cooling (CeC) 37 years ago

- **Y.S. Derbenev, Proceedings of the 7th National Accelerator Conference, V. 1, p. 269, (Dubna, Oct. 1980)**
- Coherent electron cooling, Ya. S. Derbenev, Randall Laboratory of Physics, University of Michigan, MI, USA, UM HE 91-28, August 7, 1991
- Ya.S.Derbenev, Electron-stochastic cooling, DESY , Hamburg, Germany, 1995 ……….

A principal scheme of CEC



A modification:



COHERENT ELECTRON COOLING

1. Physics of the method in general

Ya. S. Derbenev
Randall Laboratory of Physics, University of Michigan
Ann Arbor, Michigan 48109-1120 USA

UM HE 91-28

August 7, 1991

CONCLUSION

The method considered above combines principles of electron and stochastic cooling and microwave amplification. Such an unification promises to frequently increase the cooling rate and stacking of high-temperature, intensive heavy particle beams. Certainly, for the whole understanding of new possibilities thorough theoretical study is required of all principle properties and other factors of the method.

BROOKHAVEN NATIONAL LABORATORY

Stony Brook University

Why Coherent Electron Cooling ?

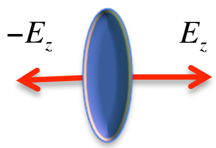
- Has potential of a rather large bandwidth W ~ 10^{13} -10^{17} Hz

$$\gamma = E_p / m_p c^2$$

- Electrons are easy to manipulate, force to radiate, bunch etc.
- **THE MOST IMPORTANT: Longitudinal electric field of bunched electron clamp is very effective way of cooling high energy hadrons – see the example below**

$$q = -e$$

- Let's assume that as result of CeC interaction a proton induced a density clamp (pancake) in the e-beam with charge of one electron



- Longitudinal electric field induced by this charge (from the Gauss law)

$$E_z = -2\pi \frac{e}{A}; \quad A = 2\pi \frac{\beta \cdot \varepsilon_n}{\gamma} - beam\ area$$

- The proton energy change in the kicker with length $L = \beta$

$$\frac{\Delta E}{E} \sim \frac{e E_z L}{\gamma m_p c^2} = -\frac{r_p}{\varepsilon_n};$$

- And cooling time will be

$$\tau \approx \frac{1}{f_o} \frac{\sigma_E}{E} \frac{\varepsilon_n}{r_p}; f_o - revolution\ frequency$$

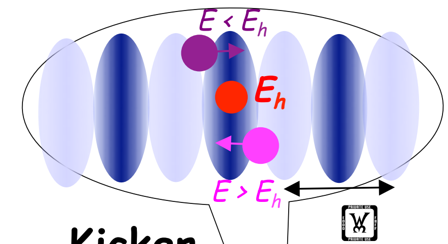
Putting parameters for 250 GeV RHIC proton beam: normalized RMS emittance of 2 mm mrad and relative energy spread of 2 x10^{-4} we get cooling time of 0.93 hours!

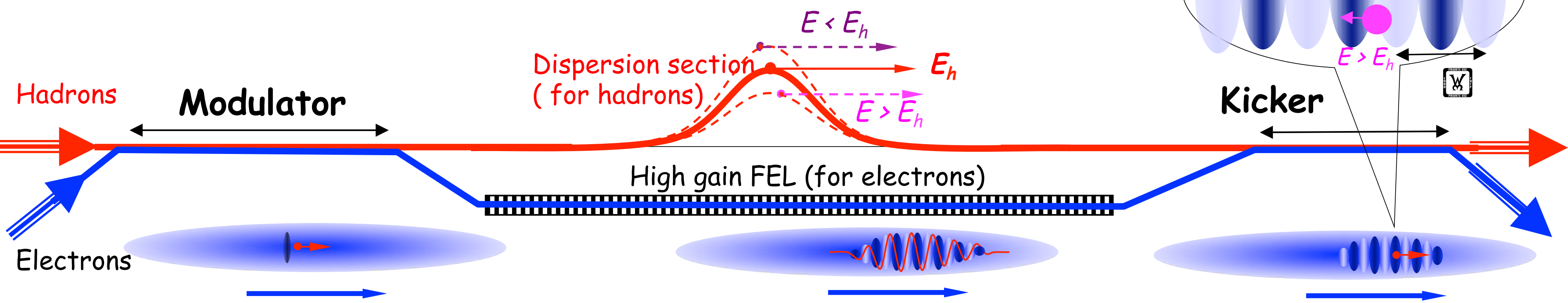
For the LHC it would be under 7 hours. Gain ~ 10 puts it under an hour.

The CeC based on the longitudinal electric field is very effective, especially when compared with using transverse fields!

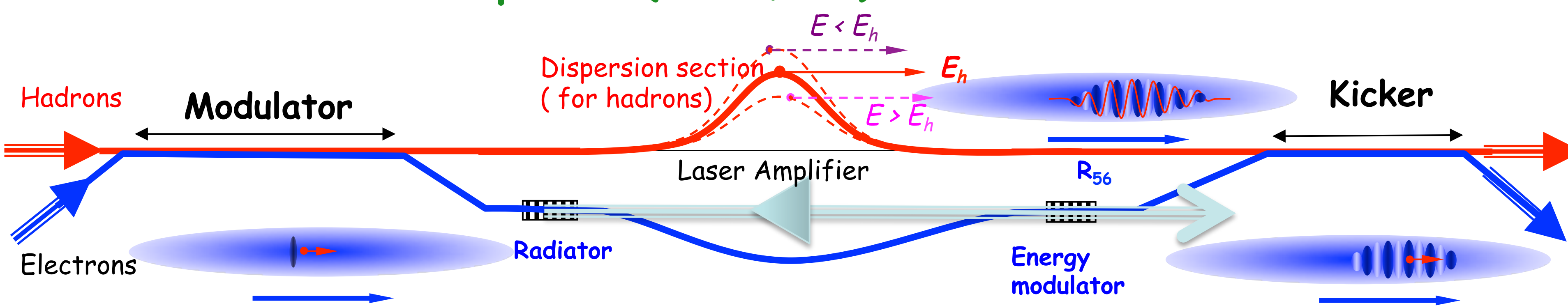
Coherent Electron Cooling Schemes

Classic – FEL amplifier (2006, VL,YD)

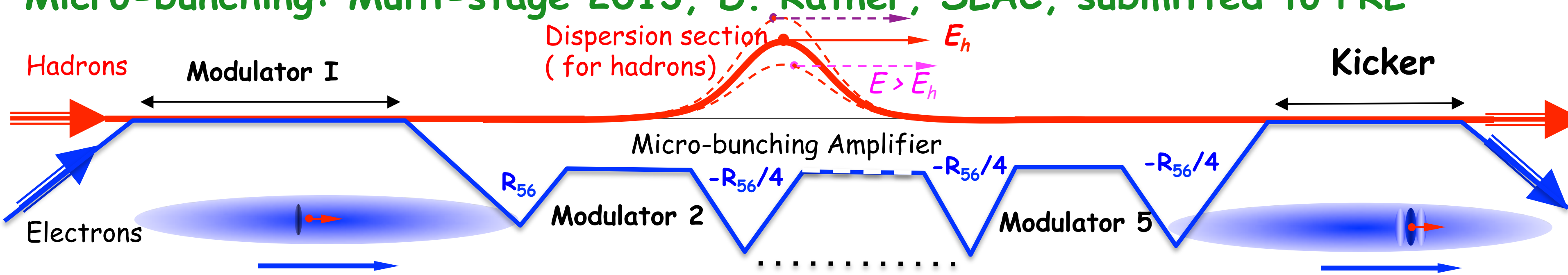




Blended – laser amplifier (2007, VL)



Enhanced bunching: single stage 2007 – VL
Micro-bunching: Multi-stage 2013, D. Ratner, SLAC, submitted to PRL



Classical CeC (γ>>1)

PRL 102, 114801 (2009) PHYSICAL REVIEW LETTERS week ending 20 MARCH 2009

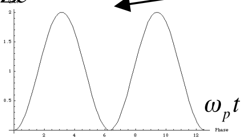
Coherent Electron Cooling

Vladimir N. Litvinenko[1,*] and Yaroslav S. Derbenev[2]

Plasma oscillation/Debye screening

$$\omega_p = \sqrt{4\pi n_e e^2/\gamma_o m_e}$$

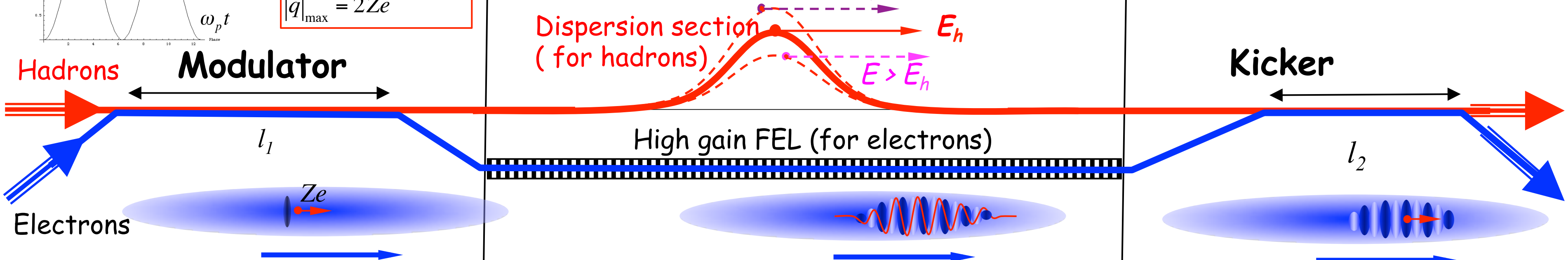
$-q/Ze$

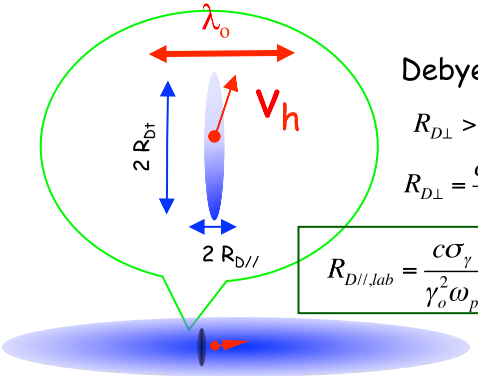


$$q = -Ze\cdot(1-\cos\varphi_1)$$
$$\varphi_1 = \omega_p l_1 / c\gamma_o$$
$$|q|_{max} = 2Ze$$

$$c\Delta t = -D\cdot\frac{\gamma-\gamma_o}{\gamma_o};\; D_{free} = \frac{L}{\gamma_o^2};\; D_{chicane} = l_{chicane}\cdot\theta^2 \ldots\ldots$$

$$\Delta E_h \approx -g_{max}\gamma_o \frac{2ZXe^2}{\pi\varepsilon_n}\cdot\frac{l_2}{\beta}\cdot\sin\left(k_o D\frac{E_h-E_o}{E_o}\right)\cdot$$

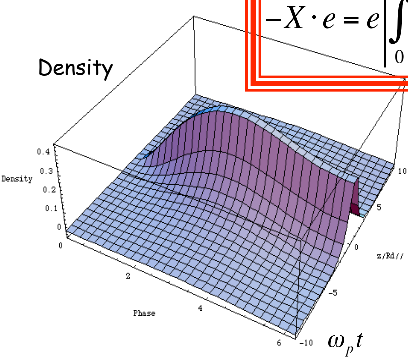




Debye radii

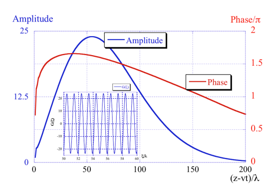
$$R_{D\perp} >> R_{D//}$$
$$R_{D\perp} = \frac{c\gamma_o\sigma_{\theta e}}{\omega_p}$$
$$R_{D//,lab} = \frac{c\sigma_\gamma}{\gamma_o^2\omega_p} << \lambda_o$$

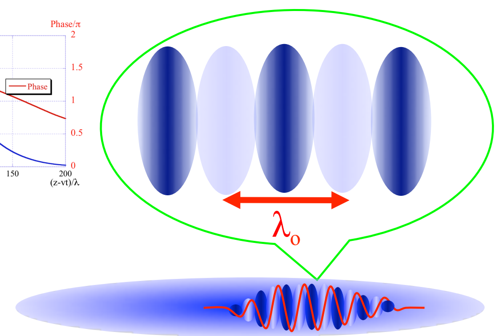
$$-X\cdot e = e\left|\int_0^{\lambda_o}\tilde n_i(z)e^{ik_{cm}z}dz\right|$$



$$X_{max} = 2Z$$

FEL Amplifier of the e-beam modulation

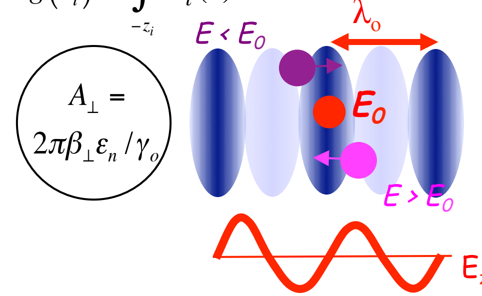




$$\lambda_o = \lambda_w\left(1+\langle\vec a_w^2\rangle\right)/2\gamma_o^2$$
$$\vec a_w = e\vec A_w / mc^2$$
$$k_o = 2\pi/\lambda_o$$
$$k_{cm} = k_o/\gamma_o$$

$$L_{Go} = \frac{\lambda_w}{4\pi\rho\sqrt{3}}$$
$$G_{FEL} = e^{(L_{FEL}-L_F)/L_G}$$
$$\Delta\varphi = \frac{L_{FEL}}{\sqrt{3}L_G}$$

$$g(z_i) = \int_{-z_i}^{\lambda_o - z_i}\tilde n_\tau(z)e^{ik_o z}dz$$



$$\Delta\varphi = 4\pi\rho \Rightarrow \varphi = -\varphi_0\cdot\cos(k_{cm}z)$$
$$\vec{\mathbf{E}} = -\vec\nabla\varphi = -\hat z\mathbf{E}_o\cdot\sin(k_{cm}z)$$
$$\rho_o(z) = Xe\frac{g(z)}{\pi\varepsilon\beta(z)\lambda_{cm}}\cos(k_{cm}z+\psi)$$
$$\mathbf{E}_o \cong Xe\frac{2g_{max}}{\pi\varepsilon\beta(z)};\quad \varepsilon\equiv\varepsilon_n/\gamma_o$$

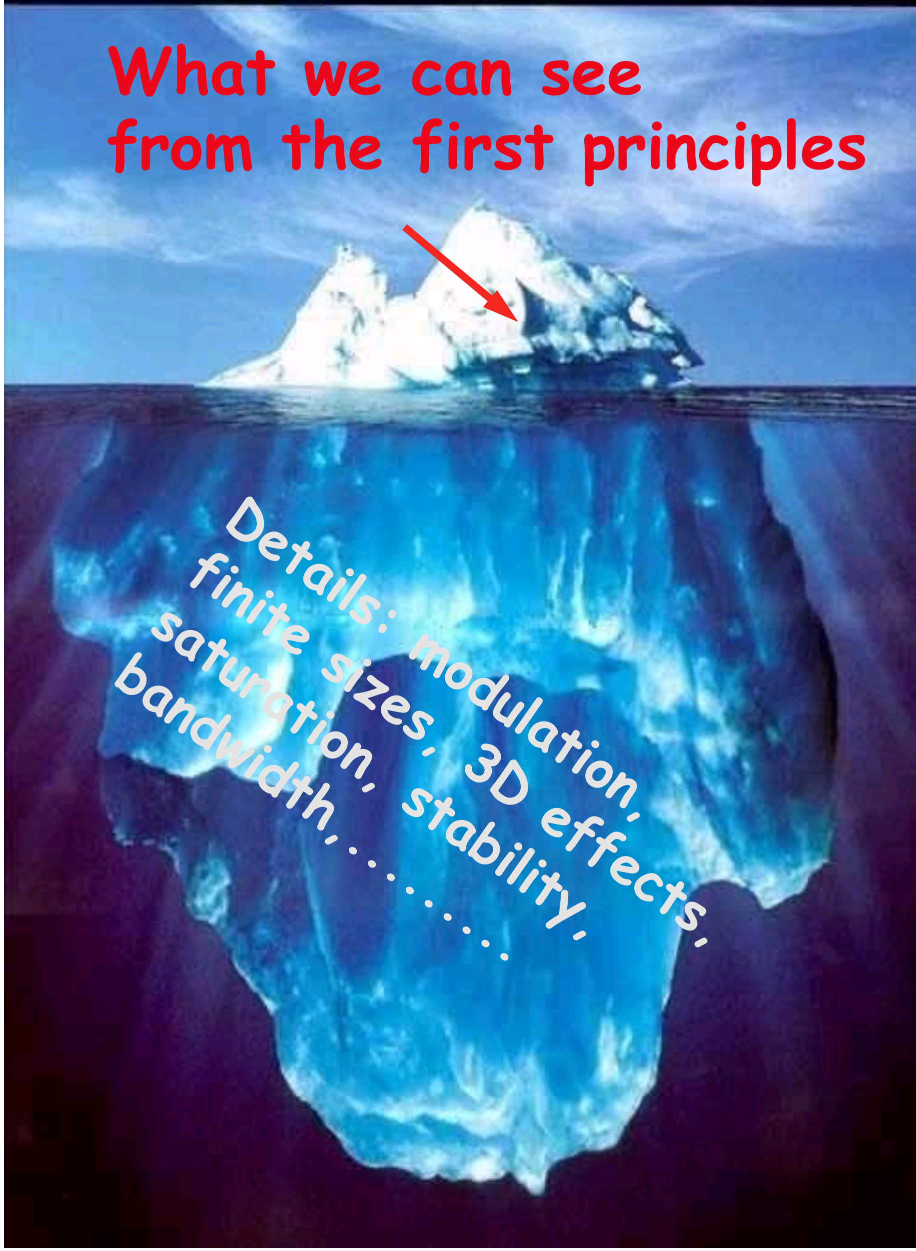
$$X \sim Z$$

CeC Parameters
Modulator

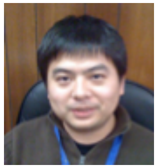
Parameter	CeC PoP	eRHIC	LHC
Spices	Au	p	p
Particles per bunch	10^9	$2x10^{11}$	$1.7x10^{11}$
Energy GeV/u	40	250	7,000
RMS ε_n, mm mrad	2.5	0.2	3
RMS energy spread	$3.7 x10^{-4}$	10^{-4}	10^{-4}
RMS bunch length, nsec	3.5	0.27	1
e-beam energy MeV	21.8	136.2	3812
Peak current	75	50	30
RMS ε_n, mm mrad	5	1	1
RMS energy spread	$1\ 10^{-4}$	$5\ 10^{-5}$	$2\ 10^{-5}$
RMS bunch length, nsec	0.05	0.27	1
Modulator length, m	**3**	**10**	**100**
Plasma phase advance, rad	**1.7**	**2.14**	**0.06**
Buncher	**None**	**None**	**Yes**
Induced charge, e	**88.1**	**1.54**	**2**

Simple on the surface

complex below the water level…

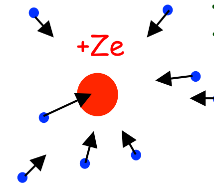


Density modulation caused by a hadron (co-moving frame)



Analytical: for kappa-2 anisotropic electron plasma,
G. Wang and M. Blaskiewicz, Phys Rev E 78, 026413 (2008)

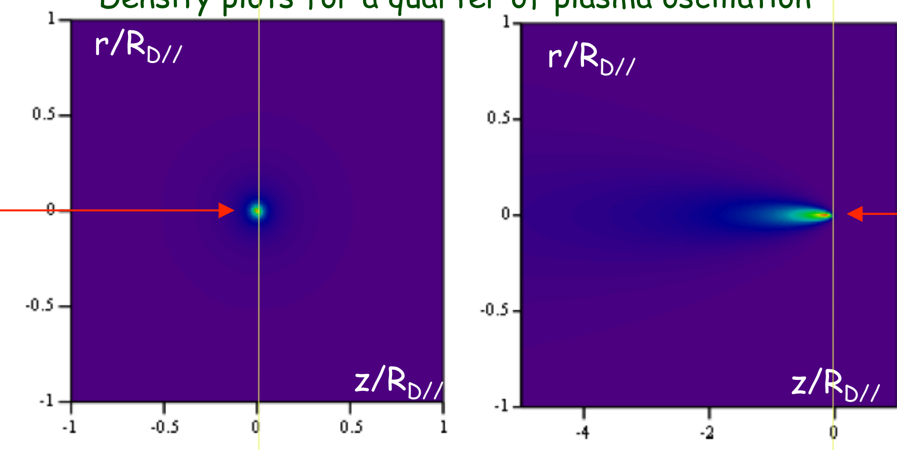
$$\tilde{n}(\vec{r},t) = \frac{Zn_o\omega_p^3}{\pi^2\sigma_{vx}\sigma_{vy}\sigma_{vz}}\int_0^{\omega_p t}\tau\sin\tau\left(\tau^2+\left(\frac{x - v_{hx}\tau/\omega_p}{r_{Dx}}\right)^2+\left(\frac{y - v_{hy}\tau/\omega_p}{r_{Dy}}\right)^2+\left(\frac{z - v_{hz}\tau/\omega_p}{r_{Dz}}\right)^2\right)^{-2}d\tau$$



Induces charge

$$q = -Ze\cdot(1-\cos\omega_p t)$$

Density plots for a quarter of plasma oscillation



Ion rests in c.m.
(0,0) is the location of the ion

Ion moves in c.m. with

$$v_{hz} = 10\sigma_{vze}$$

(0,0) is the location of the ion

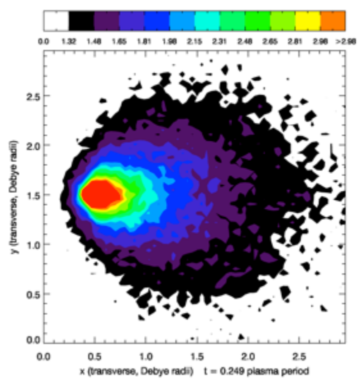


Figure 3: A transverse cross section of the wake behind a gold ion, with the color denoting density enhancement.

Numerical: VORPAL @ TechX)

Parameters of the problem

$$R_{D_\alpha} \propto \left(|v_\alpha| + \sigma_{v_\alpha}\right)/\omega_p; \quad \alpha = x,y,z$$

$$t = \tau/\omega_p; \quad \vec{v} = \vec{\nu}\sigma_{v_z}; \quad \vec{r} = \vec{\rho}\sigma_{v_z}/\omega_p; \quad \omega_p = \sqrt{\frac{4\pi e^2 n_e}{m}} \qquad s = r_{D_z} = \frac{\sigma_{v_z}}{\omega_p}$$

$$R = \frac{\sigma_{v_\perp}}{\sigma_{v_z}}; \quad T = \frac{v_{hx}}{\sigma_{v_z}}; \quad L = \frac{v_{hz}}{\sigma_{v_z}}; \quad \xi = \frac{Z}{4\pi n_e R^2 s^3};$$

$$A = \frac{a}{s}; \quad X = \frac{x_{ho}}{a}; Y = \frac{y_{ho}}{a}.$$

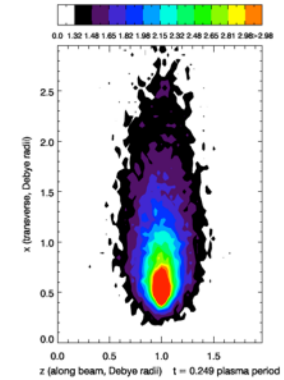


Figure 4: A longitudinal cross section of the wake behind a gold ion, with the color denoting density enhancement.

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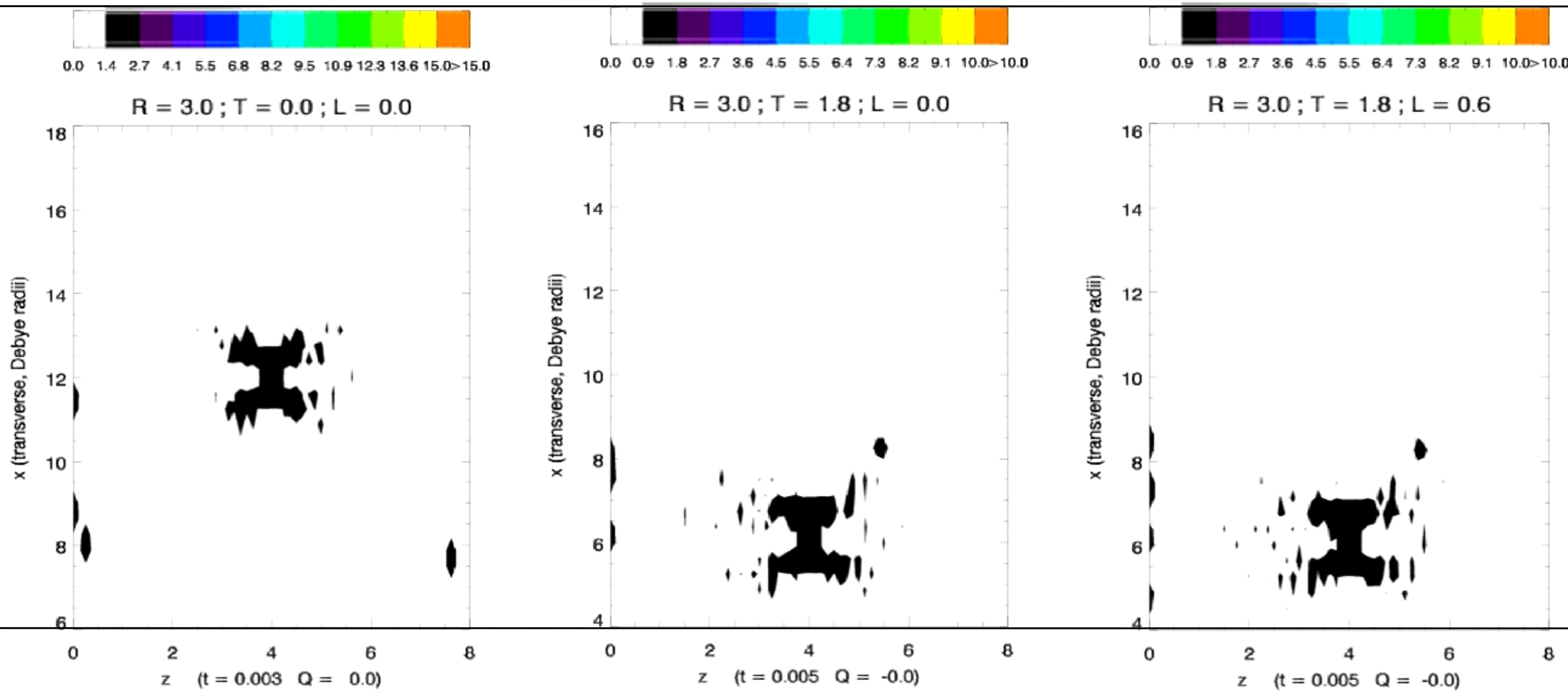
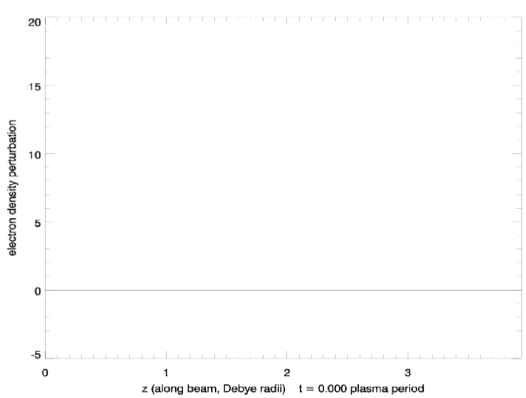
Numerical simulations (VORPAL @ TechX) Provides for simulation with arbitrary distributions and finite electron beam size

VORPAL Simulations Relevant to Coherent Electron Cooling, G.I. Bell et al., EPAC'08, (2008)



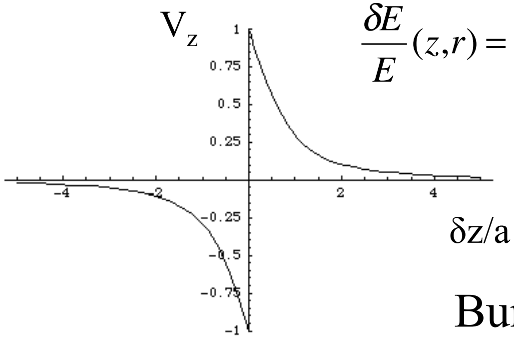


$$R = \frac{\sigma_{v_\perp}}{\sigma_{v_z}};\ T = \frac{v_{hx}}{\sigma_{v_z}};\ L = \frac{v_{hz}}{\sigma_{v_z}}$$

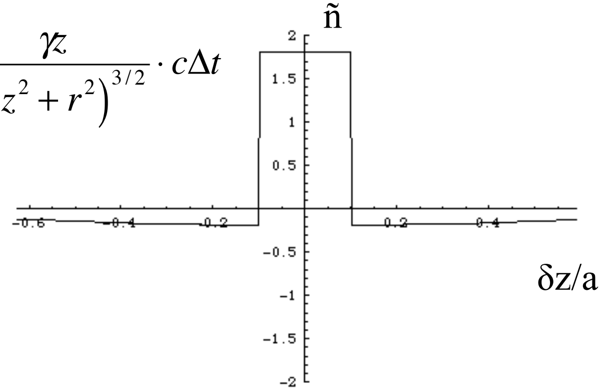
$$q = -Ze \cdot (1 - \cos \omega_p t)$$

Stony Brook University

Bunching for high energy beams ($\omega_p t << 1$)

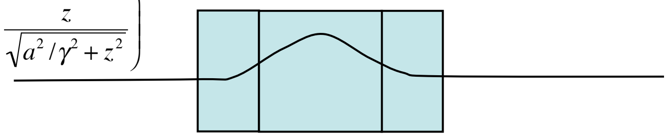


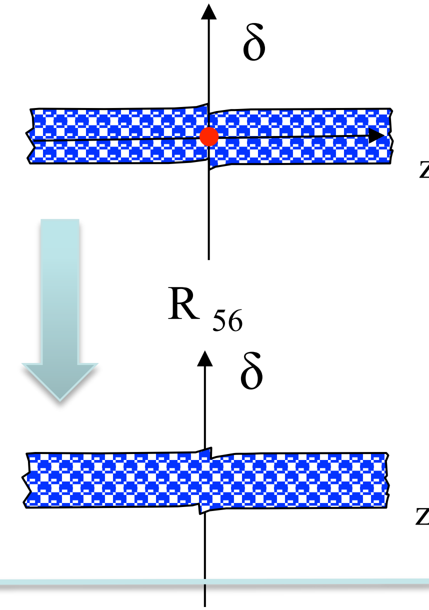
$$\frac{\delta E}{E}(z,r) = -Zr_e \frac{\gamma z}{\left(\gamma^2 z^2 + r^2\right)^{3/2}} \cdot c\Delta t$$



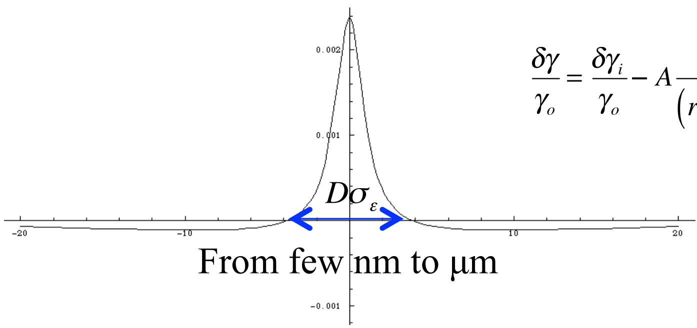
Buncher

$$\left\langle \frac{\delta E}{E} \right\rangle \cong -2Z \frac{r_e}{a^2} \cdot \frac{L_{pol}}{\gamma} \cdot \left(\frac{z}{|z|} - \frac{z}{\sqrt{a^2/\gamma^2 + z^2}} \right)$$





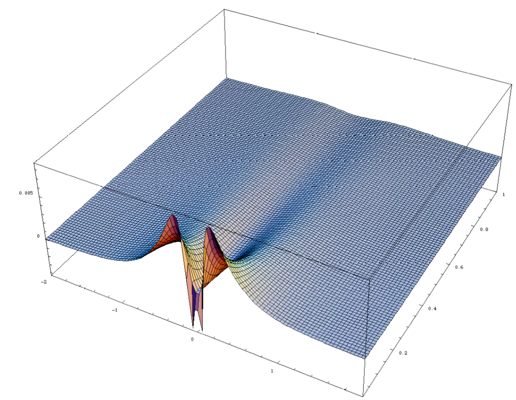
Exact calculations: solving Vlasov equation



$$\frac{\delta\gamma}{\gamma_o} = \frac{\delta\gamma_i}{\gamma_o} - A\frac{\gamma_o z_i}{\left(r_i^{\,2} + \gamma_o^{\,2} z_i^{\,2}\right)^{3/2}}; \quad z = z_i + D\left(\frac{\delta\gamma_i}{\gamma_o} - A\frac{\gamma_o z_i}{\left(r_i^{\,2} + \gamma_o^{\,2} z_i^{\,2}\right)^{3/2}} \right);$$

$$\Omega = \frac{Zr_e L}{\beta_o^2 D^2 \gamma_o^3 \sigma_\varepsilon^3}$$

$$N_e \approx 4\pi Z n_o \frac{r_e L|D|}{\beta_o^2 \gamma_o}$$



$$\tilde{\rho}(z \cdot D\sigma_\varepsilon) = \pi c_o \cdot \int_0^\infty Y\,dY \cdot \left\{ \frac{Erf\left(\frac{Y - \Omega Y^{-2} + z}{\sqrt{2}}\right) + Erf\left(\frac{Y - \Omega Y^{-2} - z}{\sqrt{2}}\right)}{1 - \Omega Y^{-3}} - Erf\left(\frac{Y + z}{\sqrt{2}}\right) - Erf\left(\frac{Y - z}{\sqrt{2}}\right) \right\};$$

For 7 TeV p in LHC CeC case: a simple "gut-feeling" estimate gave 22.9 boost in the induced charge by a buncher, while exact calculations gave 21.7. Maximum bunching depends on the e-beam quality

Modulation can be a complex phenomena both theoretically and numerically

Gaussian beam, method for general orbits, Green functions
Ion screening in confined plasma

$$n_1(\vec{x},t)=\frac{e^2}{\epsilon_0}\int_0^t\int n_1(\vec{x}',t_1)\int\frac{\partial G(\vec{x},\vec{x}')}{\partial\vec{x}}\hat{\gamma}\frac{\partial f_0}{\partial\vec{v}}\Bigg|_{\substack{\vec{x}=\vec{X}_0(t_1)\\ \vec{v}=\vec{V}_0(t_1)}} d\vec{v}\,d\vec{x}'\,dt_1+ +e\int_0^t\int\frac{\partial U_2(\vec{x},t_1)}{\partial\vec{x}}\hat{\gamma}\frac{\partial f_0}{\partial\vec{v}}\Bigg|_{\substack{\vec{x}=\vec{X}_0(t_1)\\ \vec{v}=\vec{V}_0(t_1)}} d\vec{v}\,dt_1. \quad (157)$$

$$U(\vec{x},t_1)=U_1(\vec{x},t_1)+U_2(\vec{x},t_1)= =\frac{e}{\epsilon_0}\int n_1(\vec{x}',t_1)G(\vec{x},\vec{x}')d\vec{x}'-Z\frac{e}{\epsilon_0}G(\vec{x},\vec{Y}(t_1)),$$

$$N_1(\vec{x},s)=\frac{e^2}{\epsilon_0}\int N_1(\vec{x}',s)\mathcal{L}\int\frac{\partial G(\vec{x},\vec{x}')}{\partial\vec{x}}\hat{\gamma}\frac{\partial f_0}{\partial\vec{v}}\Bigg|_{\substack{\vec{x}=\vec{X}_0(0)\\ \vec{v}=\vec{V}_0(0)}} d\vec{v}\,d\vec{x}'+ +e\mathcal{L}\int_0^t\int\frac{\partial U_2(\vec{x},t_1)}{\partial\vec{x}}\hat{\gamma}\frac{\partial f_0}{\partial\vec{v}}\Bigg|_{\substack{\vec{x}=\vec{X}_0(t_1)\\ \vec{v}=\vec{V}_0(t_1)}} d\vec{v}\,dt_1, \quad (158)$$

where $\mathcal{L}$ is a Laplace transform operator and

$$N_1(\vec{x},s)\equiv\mathcal{L}n_1(\vec{x},t) \quad (159)$$

is a Laplace image of the unknown function $n_1(\vec{x},t)$. Equation (158) is the Fredholm integral equation of the second type with a kernel

$$K(\vec{x},\vec{x}',s)=\mathcal{L}\int\frac{\partial G(\vec{x},\vec{x}')}{\partial\vec{x}}\hat{\gamma}\frac{\partial f_0}{\partial\vec{v}}\Bigg|_{\substack{\vec{x}=\vec{X}_0(0)\\ \vec{v}=\vec{V}_0(0)}} d\vec{v}, \quad (160)$$

and a left-hand side

$$F(\vec{x},s)=-Z\frac{e^2}{\epsilon_0}\mathcal{L}\int_0^t\int\frac{\partial G(\vec{x},\vec{Y}(t_1))}{\partial\vec{x}}\hat{\gamma}\frac{\partial f_0}{\partial\vec{v}}\Bigg|_{\substack{\vec{x}=\vec{X}_0(t_1)\\ \vec{v}=\vec{V}_0(t_1)}} d\vec{v}\,dt_1. \quad (161)$$

With these notations, the equation can be written in a standard form:

$$F(\vec{x},s)=N_1(\vec{x},s)-\lambda\int N_1(\vec{x}',s)K(\vec{x},\vec{x}',s)d\vec{x}', \quad (162)$$

where $\lambda=\frac{e^2}{\epsilon_0}$.

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$$G(\vec{x},\vec{x}')=-\frac{1}{2}|\vec{x}-\vec{x}'|,\qquad \frac{\partial}{\partial\vec{x}}G(\vec{x},\vec{x}')=-\frac{1}{2}\mathrm{sign}(\vec{x}-\vec{x}'), \quad (218)$$

$$G(\vec{x},\vec{x}')=-\frac{1}{2\pi}\ln|\vec{x}-\vec{x}'|,\qquad \frac{\partial}{\partial\vec{x}}G(\vec{x},\vec{x}')=-\frac{1}{2\pi}\frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^2}, \quad (219)$$

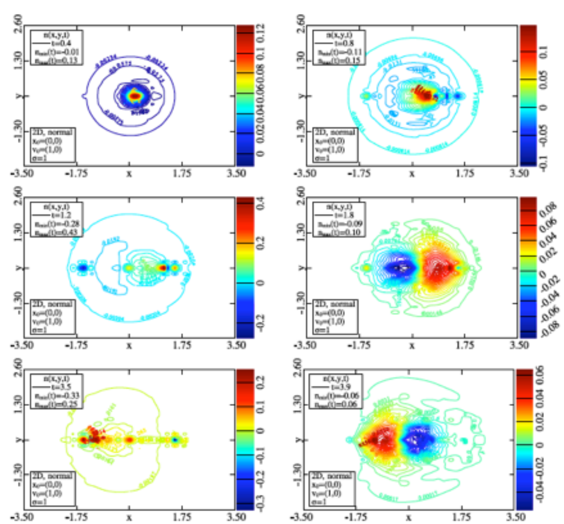
$$G(\vec{x},\vec{x}')=\frac{1}{4\pi}\frac{1}{|\vec{x}-\vec{x}'|},\qquad \frac{\partial}{\partial\vec{x}}G(\vec{x},\vec{x}')=-\frac{1}{4\pi}\frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3}, \quad (220)$$

$$K_t(\vec{x},\vec{x}',s)=\frac{e^{-ts}}{e^{\frac{2\pi}{\omega}s}-1}\int_t^{t+\frac{2\pi}{\omega}}\int R(\vec{X}_0(t_1),\vec{x}',\vec{V}_0(t_1))e^{st_1}d\vec{v}\,dt_1, \quad (225)$$

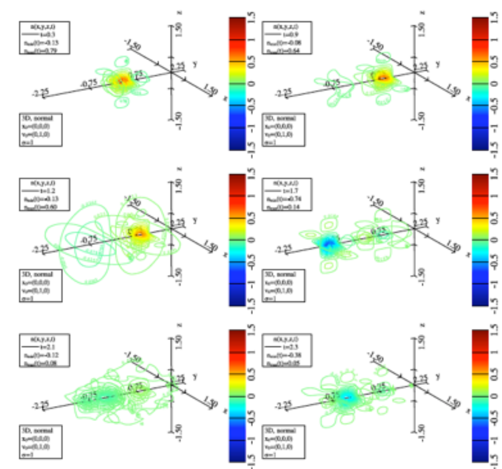
$$F_t(\vec{x},s)=-Z\frac{e^{-ts}}{e^{\frac{2\pi}{\omega}s}-1}\int_0^\infty\int_t^{t+\frac{2\pi}{\omega}}\int R(\vec{X}_0(t_1),\vec{Y}(t_1),\vec{V}_0(t_1))e^{(t_1-t_2)s}d\vec{v}\,dt_1dt_2, \quad (226)$$

Numerical methods for the Fredholm equations are very well developed using the piecewise polynomial collocation method. Solution for 2D and 3D confined plasmas

2D



3D



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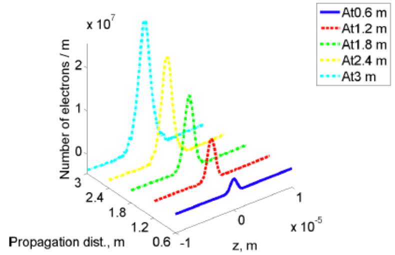
Stony Brook University

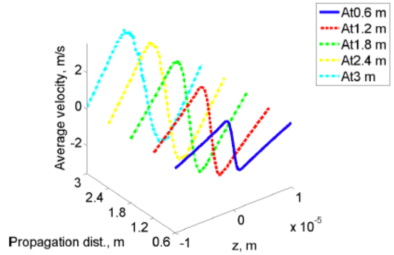
Center for Accelerator Science and Education

Modulation can be a complex phenomena with real beam in a real quadrupole channel

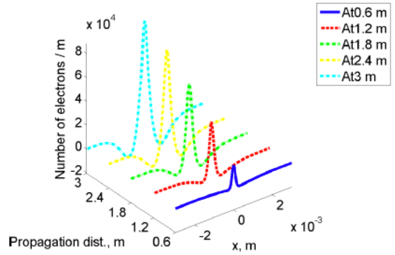
Gaussian beam in a channel with uniform focusing and compensation of space charge



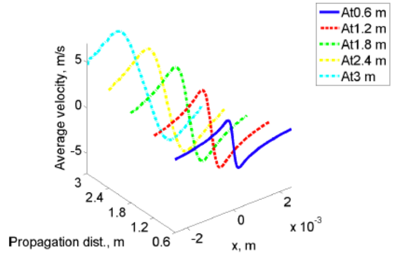
(a) Longitudinal density



(b) Longitudinal velocity

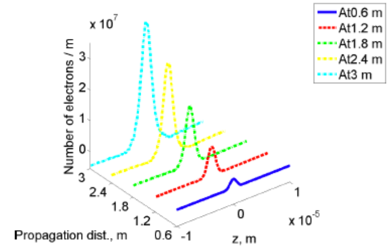


(c) Transverse density

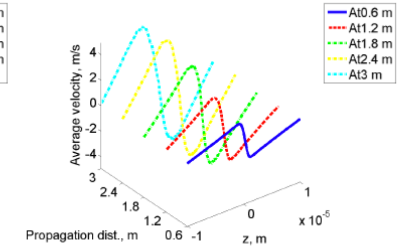


(d) Transverse velocity

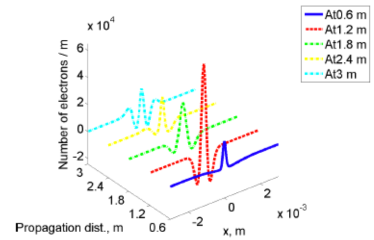
Gaussian beam in a real quadrupole channel



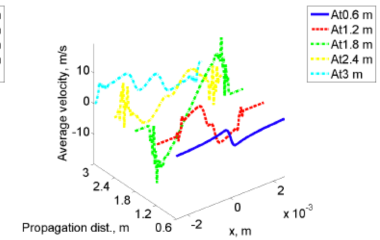
(a) Longitudinal density, 3D



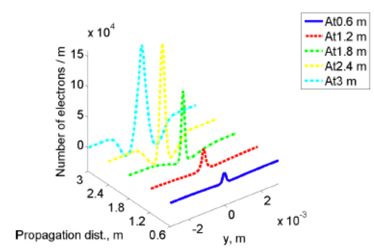
(b) Longitudinal velocity, 3D



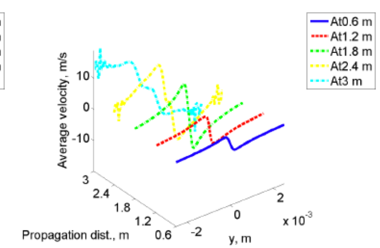
(a) Transverse (x) density, 3D



(b) Transverse (x) velocity, 3D



(c) Transverse (y) density, 3D



(d) Transverse (y) velocity, 3D

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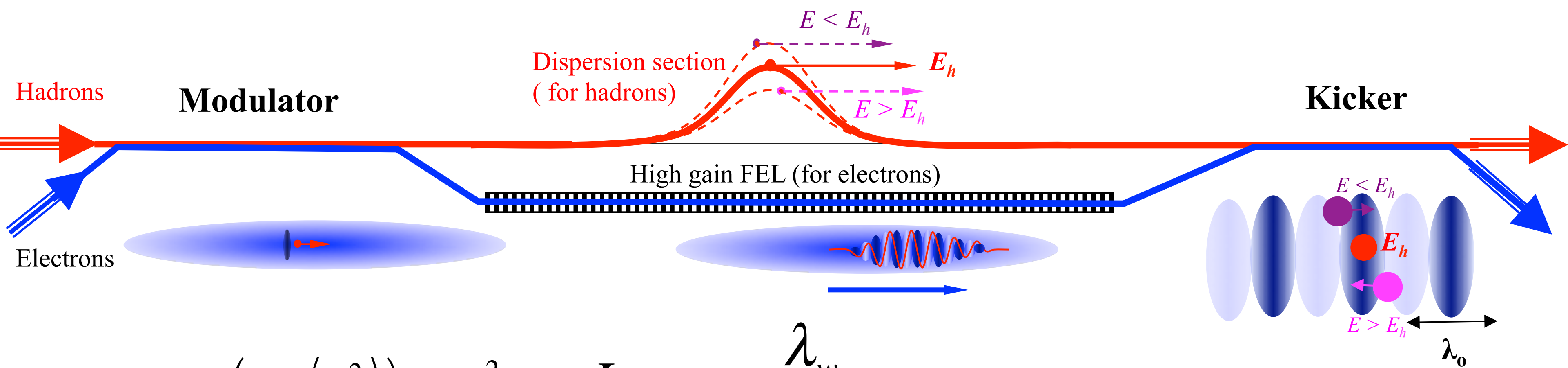
Central Section of CeC

$$D = D_{free} + D_{chicane};\ D_{free} = \frac{L}{\gamma^2};\quad D_{chicane} = l_{chicane} \cdot \theta^2$$



$$\lambda_{fel} = \lambda_w\left(1 + \left\langle \vec{a}_w^2 \right\rangle\right)/2\gamma_o^2 \qquad L_{Go} = \frac{\lambda_w}{4\pi\rho\sqrt{3}} \qquad L_G = L_{Go}(1+\Lambda)$$

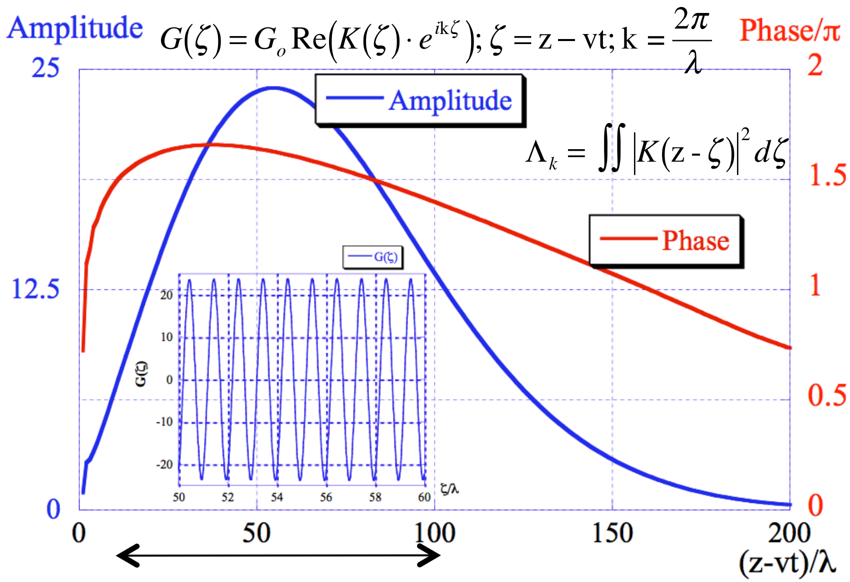
Electron density modulation is amplified in the FEL and made into a train with duration of $N_c \sim L_{gain}/\lambda_w$ alternating hills (high density) and valleys (low density) with period of FEL wavelength λ_o. Maximum gain for the electron density of High Gain FEL depends on the beam current and wavelength : for CeC experiment it can be as high as 400

$$v_{group} = (c + 2v_{//})/3 = c\left(1 - \frac{1 + a_w^2}{3\gamma^2}\right) = c\left(1 - \frac{1}{2\gamma^2}\right) + \frac{c}{3\gamma^2}\left(1 - 2a_w^2\right) = v_{hadrons} + \frac{c}{3\gamma^2}\left(1 - 2a_w^2\right)$$

3D FEL response on δ-like perturbation: Green function calculated Genesis 1.3, confirmed by RON

Example for 250 GeV protons

Energy, MeV	136.2	γ	266.45
Peak current, A	100	λ_o, nm	700
Bunchlength, psec	50	λ_w, cm	5
Emittance, norm	5 mm mrad	a_w	0.994
Energy spread	0.03%	Wiggler	Helical



The amplitude (**blue line**) and the phase (**red line** in the units of π) of the FEL gain envelope (Green function) after 7.5 gain-lengths (300 period). Total slippage in the FEL is $300\lambda_o$, λ_o=0.7 μm. A clip shows the central part of the full gain function for the range of $\zeta=\{50\lambda_o, 60\lambda_o\}$.

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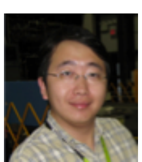
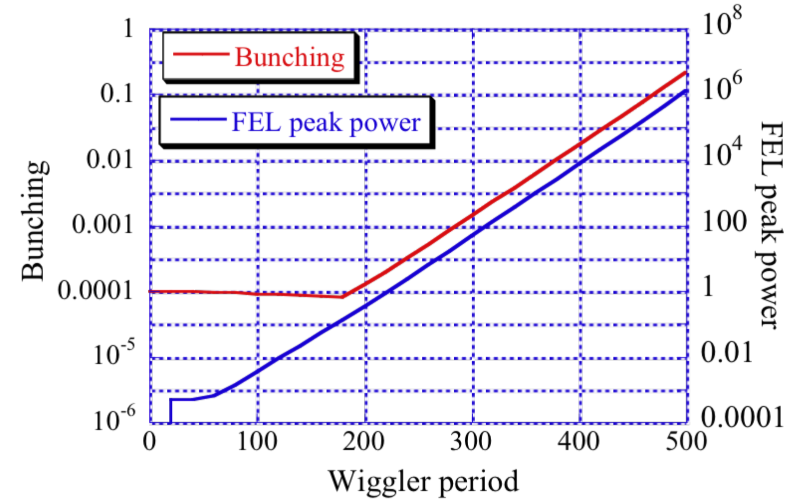
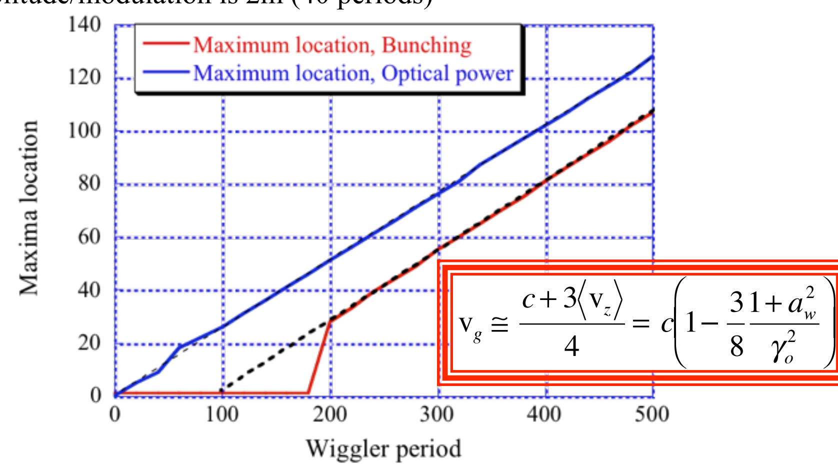




Evolution of the e-beam bunching and the FEL power simulated by Genesis. Gain length for the optical power is 1 m (20 periods) and for the amplitude/modulation is 2m (40 periods)



Propagation of the maximum of the bunching wave-packet and the FEL power simulated by Genesis, e.g. moving with group velocities. The location of the maxima, both for the optical power and the bunching progresses with a lower speed compared with prediction by 1D theory, i.e. electrons carry ~75% for the "information". There is also a delay for bunching!

What are e-beam/FEL limits ?

Gain Limitations -> Saturation

A collective instability in electron beam, including FEL or micro-bunching, is described by set of Vlasov-Maxwell equations

$$\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \vec{q}}\frac{\partial H}{\partial \vec{P}} - \frac{\partial f}{\partial \vec{P}}\frac{\partial H}{\partial \vec{q}} = 0 \qquad \frac{\partial F^{ik}}{\partial x^k} = -\frac{4\pi}{c} j^i$$

Maxwell equations a linear by definition, while Vlasov equation is not!
Hence, a model-independent estimate for maximum gain using definition of saturation when the e-beam density perturbation is in order of the initial beam density

$$\frac{\delta n}{n} \sim 1$$

The rest is a trivial (here I show 1D version) using Green-function

$$\delta n = \delta(z - z_o) \quad \Rightarrow \quad n(\tau) = n_o + \delta(z - z_o) + G_\tau(z - z_o),\ G_\tau(z) = \mathrm{Re}\, G_o(z) e^{ik_o z}$$

And assuming uncorrelated shot noise

$$n_o(0,z) = \sum_{i=1}^{N} \delta(z - z_i) \qquad \lambda_o \equiv 2\pi / k_o \qquad g(z_i) = \int_{-z_i}^{\lambda_o - z_i} G_\tau(z) e^{ik_o z}\, dz;$$

$$\left\langle \left(\frac{\delta n}{n}\right)^2 \right\rangle \sim 1 \quad \Rightarrow \quad g_{\max} \le \sqrt{\frac{I_p \cdot \lambda_o}{ec \cdot M_c}} \propto \sqrt{\frac{\delta\omega}{\omega}}$$

$$\Lambda_k = \frac{\iint |G(\zeta)|^2 d\zeta}{|G(\zeta)|^2_{\max}}; M_c = \frac{\Lambda_k}{\lambda_o}$$

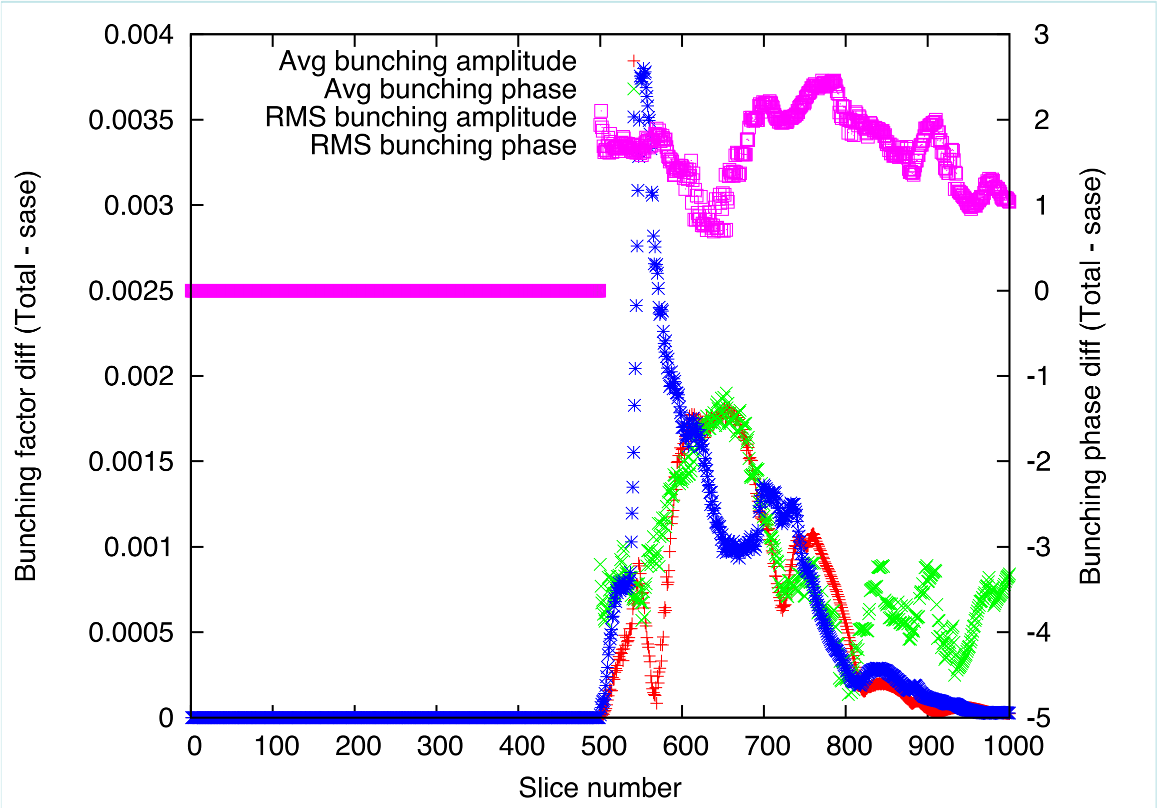
$$g_{\max} \sim 144 \cdot \sqrt{\frac{I_p[A] \cdot \lambda_o[\mu m]}{M_c}}$$

$$I_p = 10A,\ \lambda_o = 0.7\mu m;\ M_c = 38$$
$$g_{\max} \sim 62, \quad \Delta f \sim 10^{13}\ Hz$$

In excellent agreement with 3D FEL Genesis simulations

Comparing with simulation using Genesis (one example of RHIC 250 GeV p)

E_e	136 MeV
I_{peak}	10 A
ε_n	1 mm mrad
E spread	$1.5\ 10^{-5}$
λ_w	3 cm
a_w	1
λ_{fel}	422 nm
Nc	78
Δf	$1.4\ 10^{13}$ Hz
g_{max} (est)	33
g_{max} (sim)	27



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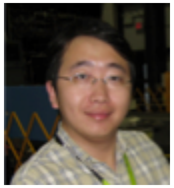
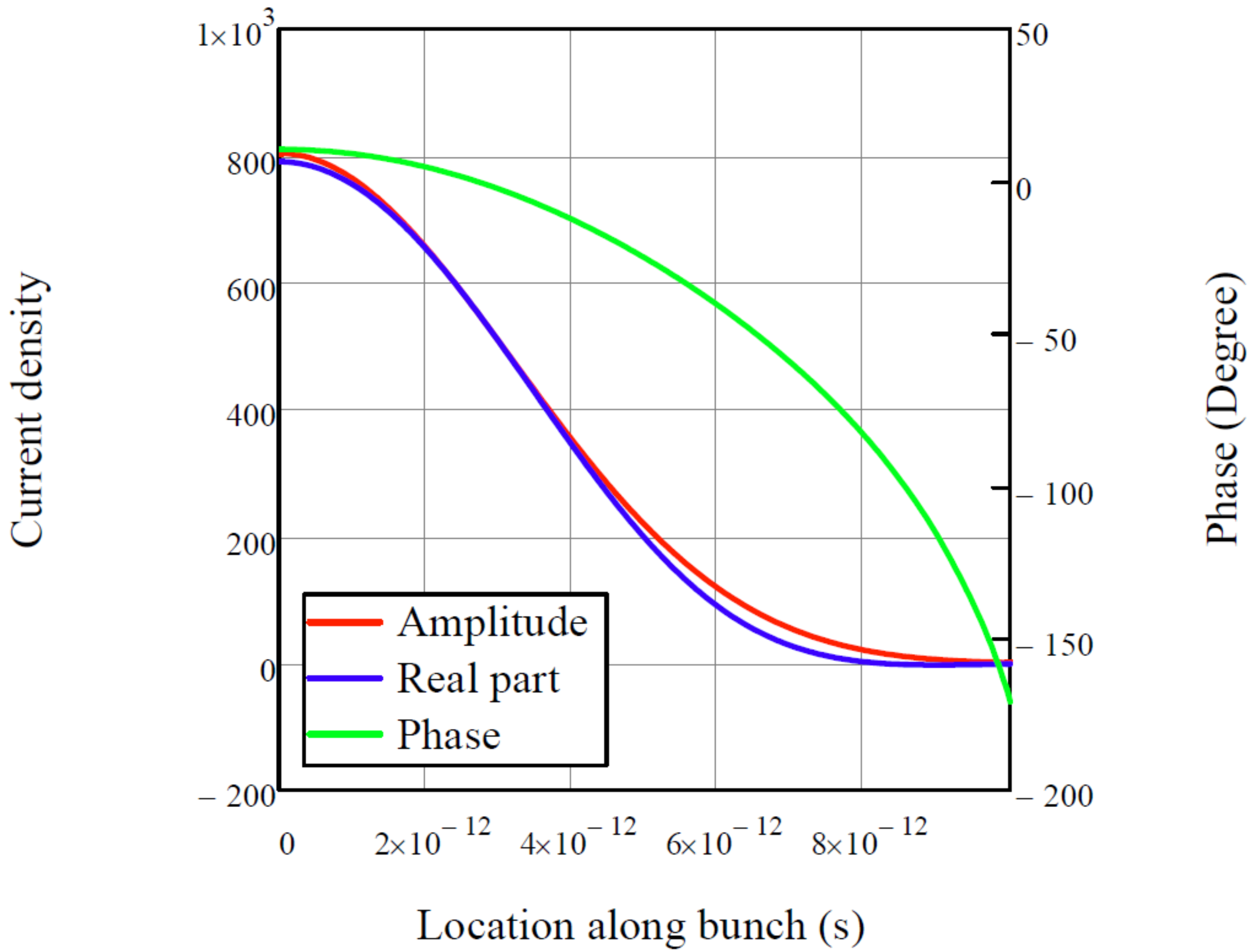


Comparison was done for 3 cases:
CeC PoP (40 GeV/u), eRHIC (250 GeV), LHC (7 TeV)

Beam conditioning for realistic beams:

Matching FEL phase velocities along the bunch



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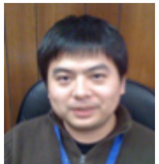


CeC Parameters: FEL amplifier

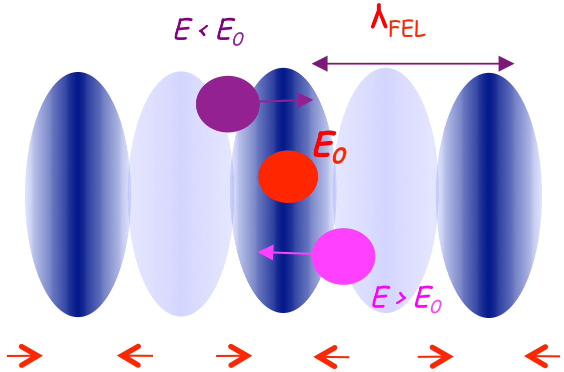
Parameter	CeC PoP	eRHIC	LHC
Spices	Au	p	p
Particles per bunch	10^9	$2x10^{11}$	$1.7x10^{11}$
Energy GeV/u	40	250	7,000
RMS ε_n, mm mrad	2.5	0.2	3
RMS energy spread	$3.7 x10^{-4}$	10^{-4}	10^{-4}
RMS bunch length, nsec	3.5	0.27	1
e-beam energy MeV	21.8	136.2	3812
Peak current	75	50	30
RMS ε_n, mm mrad	5	1	1
RMS energy spread	$1\ 10^{-4}$	$5\ 10^{-5}$	$2\ 10^{-5}$
RMS bunch length, nsec	0.05	0.27	1
λ_w, cm	**4**	**3**	**10**
λ_o, nm	**13,755**	**423**	**91**
a_w	**0.5**	**1**	**10**
g_{max}	**650**	**44**	**17**
$g_{\ required}$	**100**	**3**	**8.5**
FEL length, m	**7.5**	**9**	**100**
Bandwidth, Hz	**$6.2\ 10^{11}$**	**$1.1\ 10^{13}$**	**$2.4\ 10^{13}$**

The Kicker

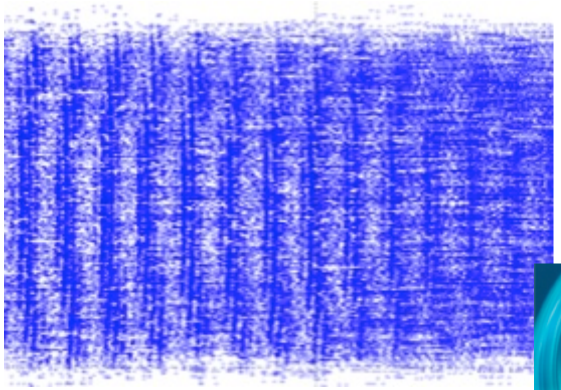
A hadron with central energy (E_o) phased with the hill where longitudinal electric field is zero, a hadron with higher energy ($E > E_o$) arrives earlier and is decelerated, while hadron with lower energy ($E < E_o$) arrives later and is accelerated by the collective field of electrons

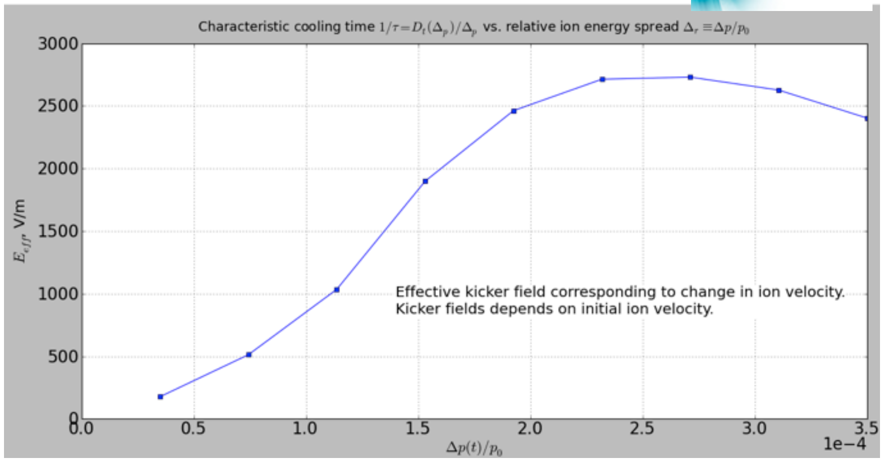
Analytical estimation

$$\Delta\varphi = 4\pi\rho \Rightarrow \varphi = -\frac{8G\cdot Ze}{\pi\beta\varepsilon_n k_{cm}}\cdot\cos(k_{cm}z);\quad \vec{\mathbf{E}} = -\vec{\nabla}\varphi = -\hat{z}\frac{8G\cdot Ze}{\pi\beta\varepsilon_n}\cdot\sin(k_{cm}z)$$



Periodical longitudinal electric field





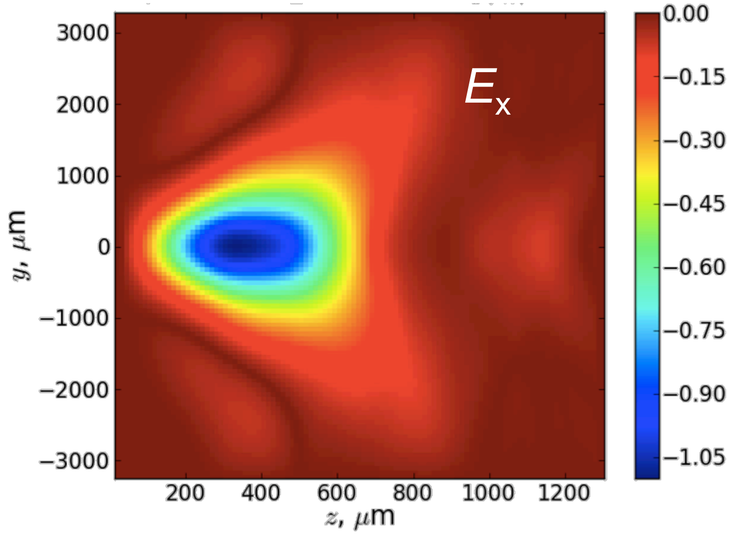
$$\frac{d\mathbf{E}}{dz} = -eE_{peak}\cdot\sin\left\{k_{fel}\cdot D_{zh}\frac{\mathbf{E}-\mathbf{E}_o}{\mathbf{E}_o}\right\};$$

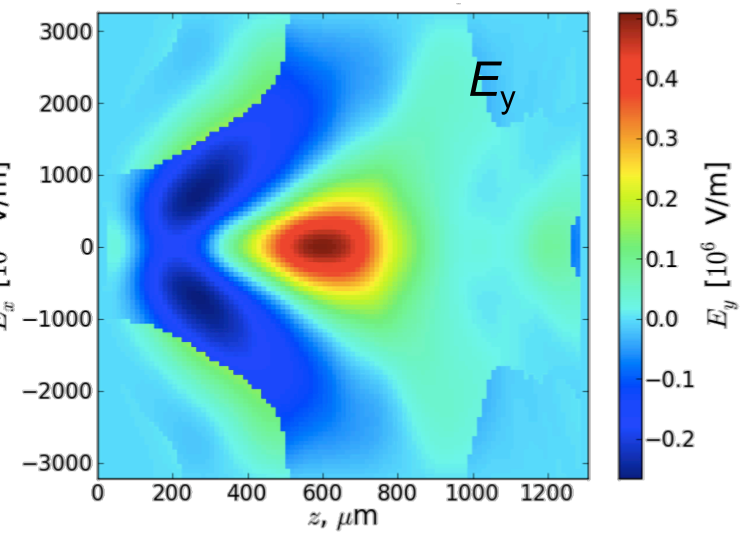
$$\zeta_{CEC} = -\frac{\Delta\mathbf{E}}{\mathbf{E}-\mathbf{E}_o} \approx \frac{e\cdot E_o\cdot l_2}{\gamma_o m_p c^2\cdot\sigma_\varepsilon}\cdot\frac{Z^2}{A}$$

$$\chi = k_{fel}D_{zh}\sigma_{\delta h}\sim 1;\quad \sigma_{\delta h} = \frac{\sigma_{\mathbf{E}}}{\mathbf{E}_o}$$

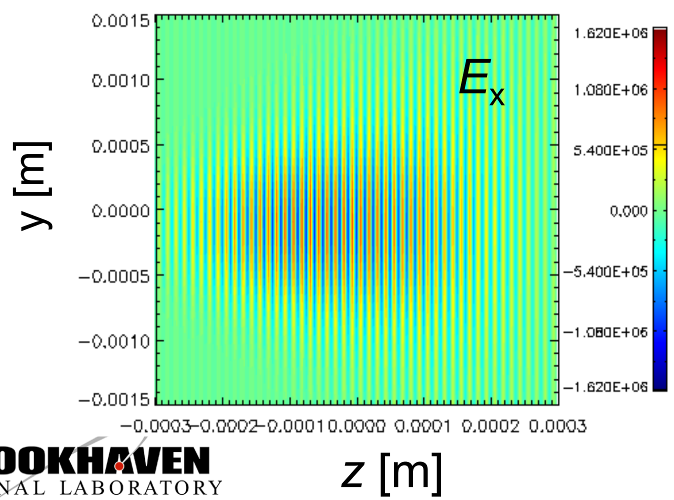
FEL electric fields can be coupled correctly from GENESIS to VORPAL in the lab frame

GENESIS output:

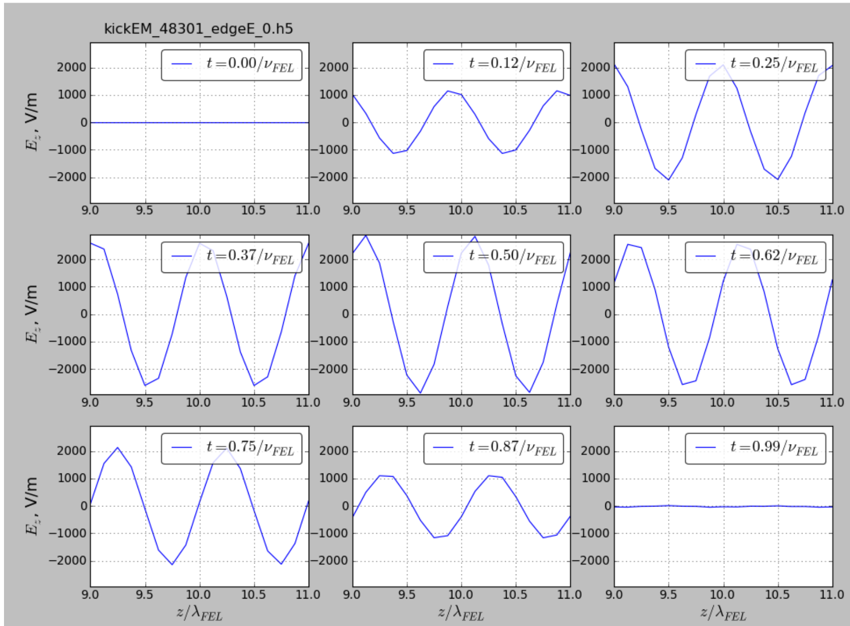




GENESIS outputs only E_x & E_y envelopes for FEL field. In VORPAL, fast oscillations are added; then E_z evolves self-consistently:



Longitudinal E-field



Analytical formula for damping decrement when e-bunch is shorter than the hadron bunch

$$\langle \zeta_{CeC} \rangle = \zeta \frac{\sigma_{\tau,e}}{\sigma_{\tau,h}} = ff \cdot 2G_o \cdot \frac{Z^2}{A} \cdot \frac{r_p \cdot \sigma_{\tau,e}}{\varepsilon_{\perp n}\left(\sigma_\delta \cdot \sigma_{\tau,h}\right)}; \; ff \sim 1$$

$$\langle \zeta_{CeC} \rangle \sim \frac{1}{\varepsilon_{long,h}\varepsilon_{trans,h}}$$

a) Does not depend on the energy of particles

b) Improves as cooling goes on

"He must have forgotten something"

Transverse size effects

$$\rho(\vec{r}) = \rho_o(r)\cdot\cos(kz);$$

$$\Delta\varphi = 4\pi\rho \Rightarrow \varphi(\vec{r}) = \varphi_o(r)\cdot\cos(kz);$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\varphi_o}{dr}\right) - k^2\varphi_o = 4\pi\rho_o(r)$$

$$\varphi(\vec{r}) = -4\pi\cos(kz)\left\{I_0(kr)\int_r^\infty \xi K_0(k\xi)\cdot\rho_o(\xi)d\xi + K_0(kr)\int_0^r \xi I_0(k\xi)\cdot\rho_o(\xi)d\xi\right\}$$

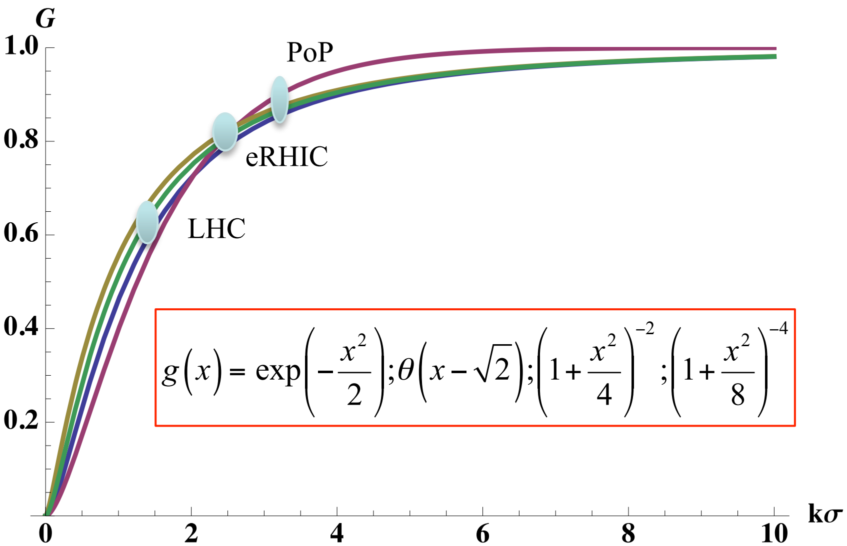
$$E_z = -\frac{\partial\varphi}{\partial z} = -4\pi k\sin(kz)\left\{I_0(kr)\int_r^\infty \xi K_0(k\xi)\cdot\rho_o(\xi)d\xi + K_0(kr)\int_0^r \xi I_0(k\xi)\cdot\rho_o(\xi)d\xi\right\}$$

$$E_r = -\frac{\partial\varphi}{\partial r} = 4\pi k\cos(kz)\left\{I_1(kr)\int_r^\infty \xi K_0(k\xi)\cdot\rho_o(\xi)d\xi - K_1(kr)\int_0^r \xi I_0(k\xi)\cdot\rho_o(\xi)d\xi\right\}$$

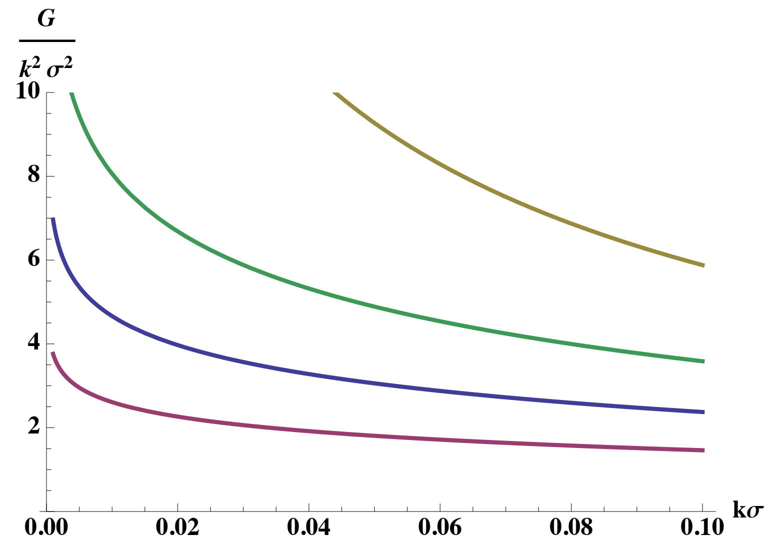
$$\rho(r) = \rho(0)\cdot g(r/\sigma)$$

$$E_{zo}(r=0) \propto -\frac{4\pi\tilde{q}}{\sigma^2}G(k_{cm}\sigma)$$

$$k_{cm}\sigma_\perp = \frac{k_o}{\gamma_o}\sqrt{\frac{\beta_\perp\varepsilon_{n\perp}}{\gamma_o}} = \sqrt{\gamma_o}\sqrt{\beta_\perp\varepsilon_{n\perp}}\,\frac{k_w}{2(1+a_w^2)}$$



$$g(x) = \exp\left(-\frac{x^2}{2}\right);\theta\left(x-\sqrt{2}\right);\left(1+\frac{x^2}{4}\right)^{-2};\left(1+\frac{x^2}{8}\right)^{-4}$$



Effects of the surrounding

Stochasticity vs coherence

Effects of the surrounding particles

Each charged particle causes generation of an electric field wave-packet proportional to its charge and synchronized with its initial position in the bunch

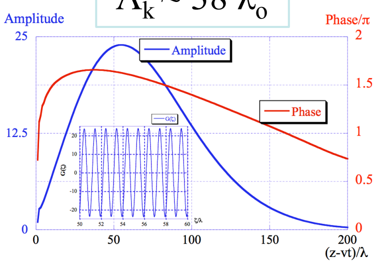
$$\mathbf{E}_{total}(\zeta) = \mathrm{E_o} \cdot \mathrm{Im}\left(X \cdot \sum_{i,hadrons} K(\zeta - \zeta_i) e^{ik(\zeta - \zeta_i)} - \sum_{j,electrons} K(\zeta - \zeta_j) e^{ik(\zeta - \zeta_j)} \right)$$

Evolution of the RMS value resembles stochastic cooling!
Best cooling rate achievable is $\sim 1/N_{eff}$, N_{eff} is effective number of hadrons in coherent sample ($\Lambda_k = M_c \lambda_o$)

$$\left\langle \delta^2 \right\rangle' = -2\xi \left\langle \delta^2 \right\rangle + D$$

$$\Lambda_k = \iint \left| K(z - \zeta) \right|^2 d\zeta$$

$\Lambda_k \sim 38\, \lambda_o$



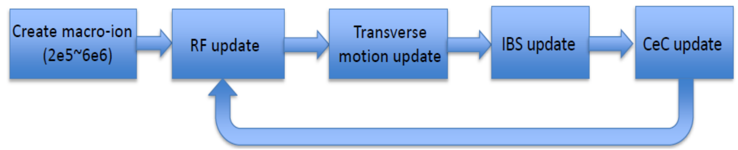
$$N_{eff} \cong N_h \frac{\Lambda_k}{\sqrt{4\pi}\sigma_{z,h}} + \frac{N_e}{X^2} \frac{\Lambda_k}{\sqrt{4\pi}\sigma_{z,e}}$$

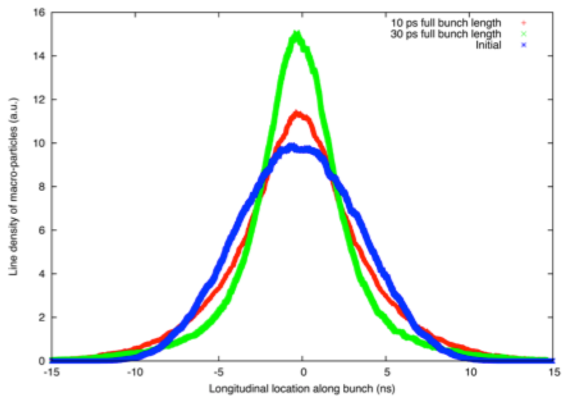
$$\xi = -g \left\langle \delta_i \, \mathrm{Im}\left(K(\Delta\zeta_i) e^{ik\Delta\zeta_i} \right) \right\rangle / \left\langle \delta^2 \right\rangle; \; D = g^2 N_{eff} / 2;$$

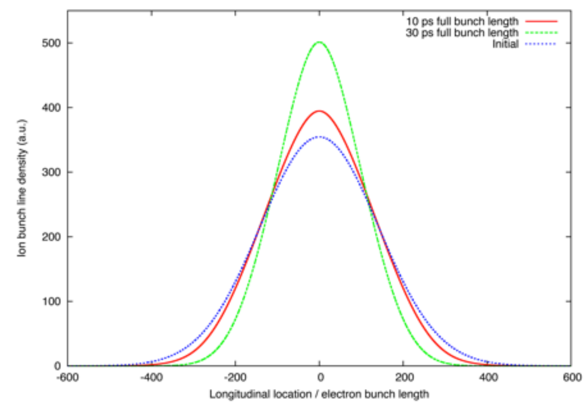
$$g \cong g_{\max} \frac{Z \cdot X}{A} \frac{2 r_p}{\pi \varepsilon_{\perp n}} f(\varphi_2) \cdot \frac{l_2}{\beta},$$

$$\xi_{CeC}(\max) = \frac{\Delta}{2\sigma_\gamma} = \frac{2}{N_{eff}} (kD\sigma_\varepsilon) \propto \frac{1}{N_{eff}}$$

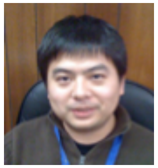
CeC experiment - simulations







The ion bunch longitudinal profiles after 40 minutes of cooling.
Left - the ion bunch profiles as obtained from macro-ion tracking;
Right - ion bunch profiles as obtained from numerically solving Fokker-Planck equation.

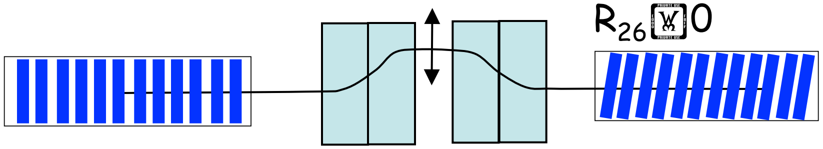


How to cool transversely

Transverse cooling

- Transverse cooling can be obtained by using coupling with longitudinal motion via transverse dispersion
- Sharing of cooling decrements is similar to sum of decrements theorem for synchrotron radiation damping, i.e. decrement of longitudinal cooling can be split into appropriate portions to cool both transversely and longitudinally: $J_s+J_h+J_v=1$
- Vertical (better to say the second eigen mode) cooling is coming from transverse coupling

Non-achromatic chicane installed at the exit of the FEL before the kicker section turns the wave-fronts of the charged planes in electron beam



$$\delta(ct) = -R_{26}\cdot x$$

$$\Delta \mathbf{E} = -eZ^2\cdot E_o\cdot l_2\cdot \sin\left\{k\left(D\frac{\mathbf{E}-\mathbf{E}_o}{\mathbf{E}_o} + R_{16}x' - R_{26}x + R_{36}y' + R_{46}y\right)\right\};$$

$$\Delta x = -D_x\cdot eZ^2\cdot E_o\cdot L_2\cdot kR_{26}x + ...$$

$$\zeta_\perp = J_\perp \zeta_{CeC};\quad \zeta_{//} = (1-2J_\perp)\zeta_{CeC};$$

$$\frac{d\varepsilon_x}{dt} = -\frac{\varepsilon_x}{\tau_{CeC\perp}};\ \frac{d\sigma_\varepsilon^{\ 2}}{dt} = -\frac{\sigma_\varepsilon^{\ 2}}{\tau_{CeC\,//}}$$

$$\tau_{CeC\perp} = \frac{1}{2J_\perp\zeta_{CeC}};\ \tau_{CeC\perp} = \frac{1}{2(1-2J_\perp)\zeta_{CeC}};$$

Distribution of the decrements

$$X=\frac{1}{2}\sum_{k=1}^{3}\left(a_kY_k(s)e^{i\psi_k}+c.c.\right);\;\; Y_j^{*T}SY_k=2i\delta_{jk}; Y_j^{T}SY_k=0;\;\; S=\begin{bmatrix}\sigma&0&0\\0&\sigma&0\\0&0&\sigma\end{bmatrix};\sigma=\begin{bmatrix}0&1\\-1&0\end{bmatrix}$$

$$\delta X=-\xi\begin{bmatrix}0\\0\\0\\0\\0\\\delta+k_xx\end{bmatrix}=-\xi K\cdot X=-\xi\begin{bmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\k_x&0&0&0&0&1\end{bmatrix}\cdot\frac{1}{2}\sum_{k=1}^{3}\left(a_kY_k(s)e^{i\psi_k}+c.c.\right);$$

$$\delta a_k=-\xi\frac{e^{-i\psi_k}}{2i}Y_k^{*T}SK\cdot\sum_{j=1}^{3}\left(a_jY_j(s)e^{i\psi_j}+c.c.\right);$$

$$\xi_k=\frac{\langle\delta a_k\rangle}{a_k}=-\xi\frac{Y_k^{*T}SKY_k}{2i};\qquad 2\cdot\sum_{k=1}^{3}\xi_k=\xi\cdot Tr(K)=\xi;$$

$$\xi_k=\frac{\xi}{2i}\cdot Y_k^{5*}\left(k_xY_k^1+Y_k^6\right)$$

$$X^T=\{x,x',y,y',-c\tau,\delta\}$$

$$k_x=\frac{R_{52e}}{D_{zh}}$$

Distribution of the decrements

$$Q_s << Q_{1,2}$$

$$Y_{k=1,2} \cong \begin{pmatrix} Y_{k1} \\ Y_{k2} \\ Y_{k3} \\ Y_{k4} \\ Y_{k5} \\ 0 \end{pmatrix} = \begin{pmatrix} Z_k \\ -Z_k^T SD \\ 0 \end{pmatrix}; Y_3 \cong \frac{1}{\sqrt{\Omega}} \begin{pmatrix} D_x \\ D'_x \\ D_y \\ D'_y \\ i\Omega \\ 1 \end{pmatrix};$$

$$\xi_k = \frac{\xi}{2i} \cdot Y_k^{*5} \left(k_x Y_k^1 + Y_k^6 \right)$$

$$\xi_s = \frac{\xi}{2} \left(k_x D_x + 1 \right);$$

$$\xi_{k=1,2} = -\frac{\xi}{2i} \cdot \left(Z_k^{*T} SD \right) \cdot k_x Z_k^1$$

$$\xi_1 + \xi_2 = -k_x D_x \frac{\xi}{2}$$

Uncoupled case

$$\xi_y = 0; \quad \mathrm{Re}\,\xi_x = -\frac{\xi}{2} \cdot R_{52e} \frac{D_{xh}}{D_{zh}}; \quad \mathrm{Re}\,\xi_s = \frac{\xi}{2} \cdot \left(1 - R_{52e} \frac{D_{xh}}{D_{zh}} \right)$$

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Typical mix: full coupling, three equal decrements

$$R_{52e} \sim 10^{-3};\ D_{zh} \sim \frac{\lambda_{FEL}}{2\pi\sigma_{\delta h}} \Rightarrow D_{xh} \sim \frac{2}{3}\cdot\frac{\lambda_{FEL}}{2\pi\sigma_{\delta h}} \sim 10^{2}\cdot\frac{\lambda_{FEL}}{\sigma_{\delta h}} \sim 10^{5}\,\lambda_{FEL}$$

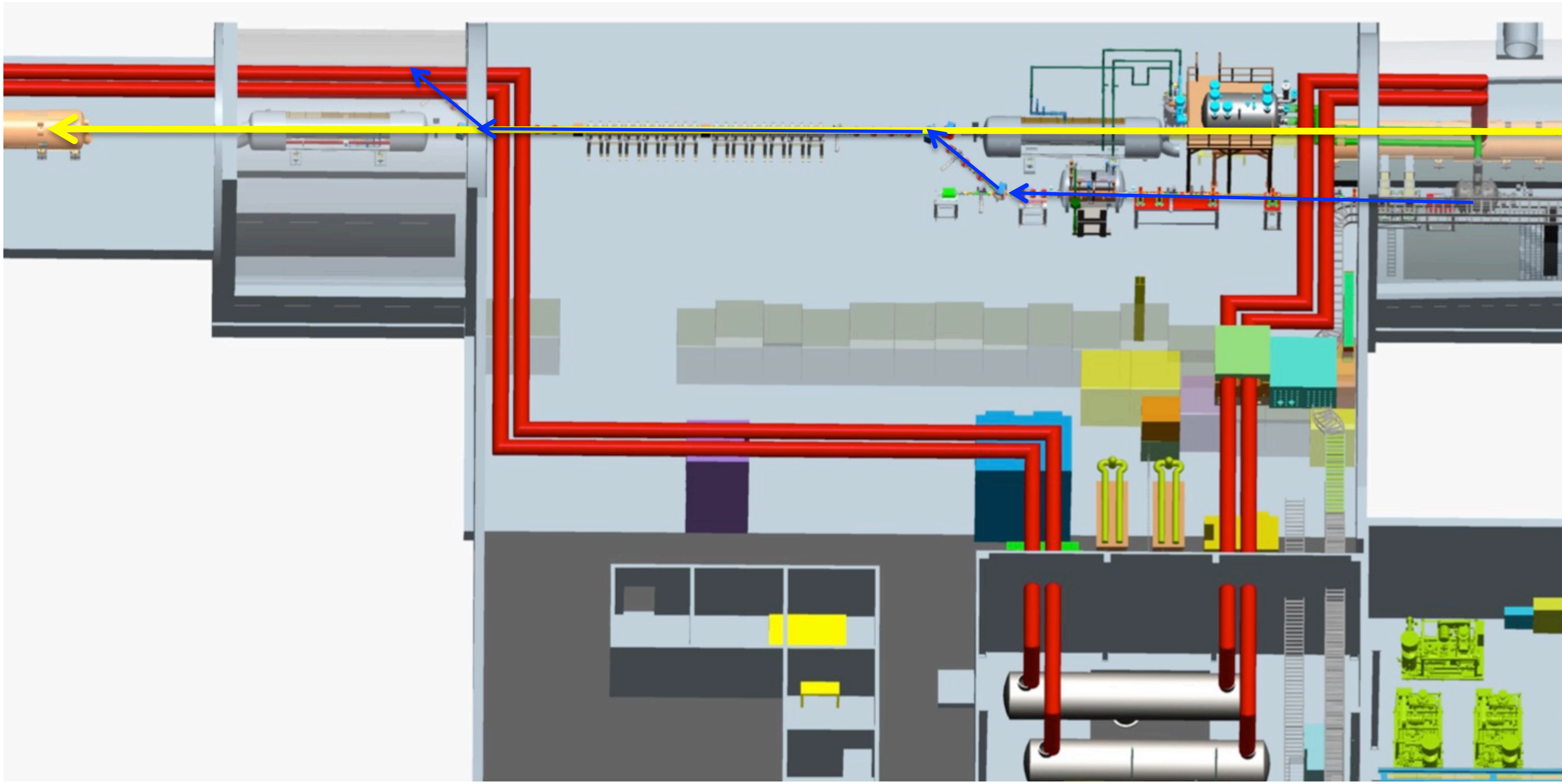
CeC PoP would need

$$\lambda_{FEL} \sim 10^{-5} \Rightarrow D_{xh} \sim 1m$$

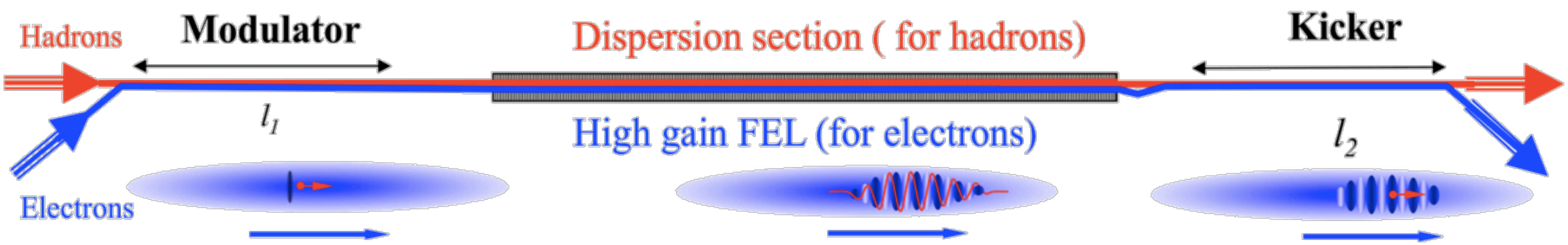
eRHIC cooler

$$\lambda_{FEL} \sim 0.5\cdot 10^{-6} \Rightarrow D_{xh} \sim 0.05m$$

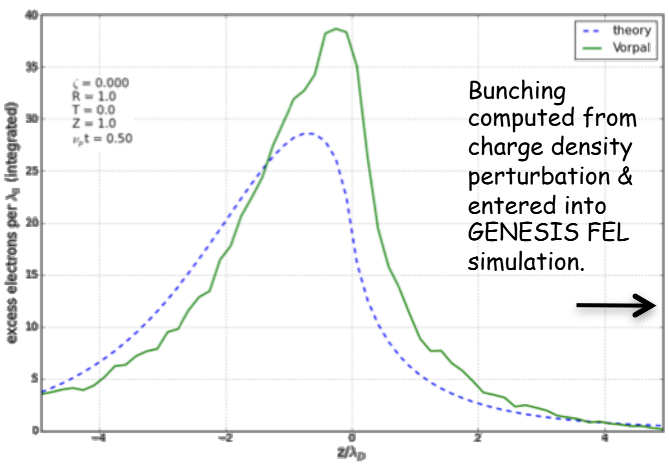
CeC Proof-of-Principle Experiment
40 GeV/u Au ions cooled by 22 MeV electrons



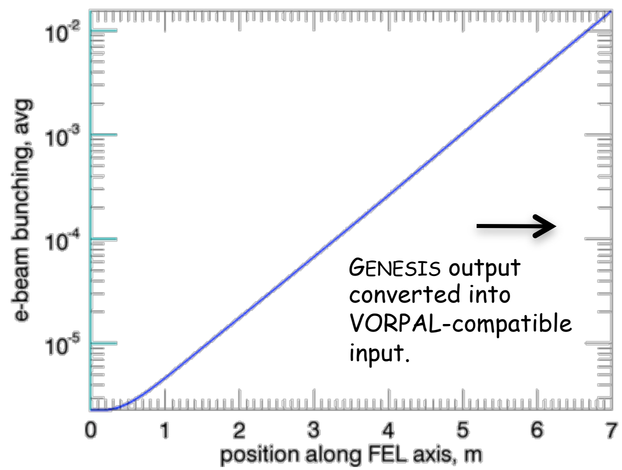
Coherent *e*-Cooling Performance Simulation with VORPAL & GENESIS



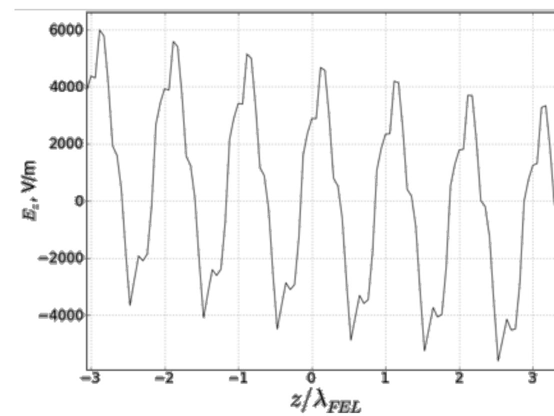
Param.'s from 40 GeV proof-of-principle exp. at BNL



VORPAL 3D δf PIC computation of e- density perturbation near Au^{+79} ion (green) vs. idealized theory (blue). On Cray XE6 cluster at NERSC.



GENESIS parallel computation of electron beam bunching in free electron laser (FEL) shows amplification of modulator signal.



VORPAL prediction of the coherent kicker electric field E_k due to e-density perturbation from modulator, amplified in the FEL.

Simulations by Tech-X

Conclusions

- **At the moment there are two methods promising cooling of dense high-energy hadron beams – optical stochastic and coherent electron cooling**
- **In my opinion the later is more versatile and promises to deliver bandwidth exceeding that of optical stochastic cooling by orders of magnitude**
- **Test of the coherent electron cooling is progress at our department –** ***details are in next presentation, CeC Part 2***
- **Novel CeC schemes are under development with promises going far beyond the classical CeC**
- **There is a lot of other fascinating (and frequently very tough problem) things we found working on CeC – too much to discuss in a single talk – and it is perfect set of subjects for master and PhD students to grasp complexity and excitement of modern accelerator and plasma physics**

Q&E