1. The energy loss per turn is given by

$$U_0 = \frac{e^2 \beta^3 \gamma^4}{3\varepsilon_0 \rho} \ . \tag{1}$$

With $\rho = 7m$ and $\gamma = 5GeV / 0.511MeV = 9785$ , eq. (1) yields

$$U_0 = \frac{e^2 \beta^3 \gamma^4}{3\varepsilon_0 \rho} = 7.899MeV = 1.266\times10^{-12} J \ . \tag{2}$$

The critical photon energy is given by

$$E_c = \hbar \omega_c , \tag{3}$$

where $\hbar$ is the denoted Planck constant and

$$\omega_c = \frac{3}{2}\gamma^3 \frac{c}{\rho} \approx 6.018\times10^{19} rad / s \tag{4}$$

is the critical angular frequency of the synchrotron radiation. Inserting eq. (4) into eq. (3) yields

$$E_c \approx 39.61KeV = 6.347\times10^{-15} J \ . \tag{5}$$

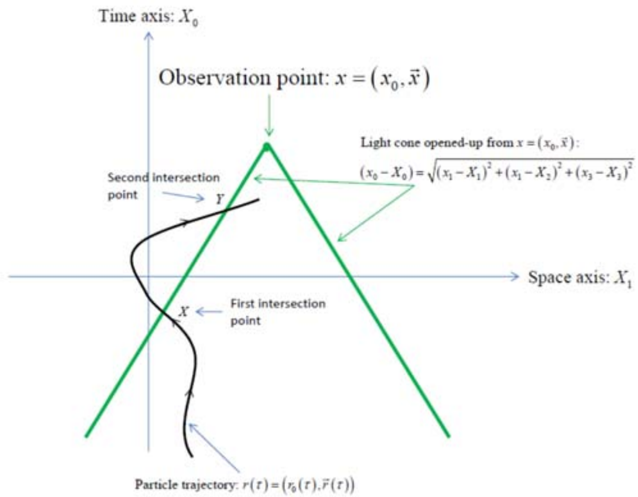
The total synchrotron radiation power for a beam is given by the 1-turn energy loss of all particles in the ring divided by the time it takes for one circulation (i.e. the revolution period)

$$P_{beam} = \left(U_0 \cdot N_{ring}\right)\frac{1}{T_{rev}} = \left(U_0 \cdot \frac{I_b}{e} T_{rev}\right)\frac{1}{T_{rev}} = U_0 \frac{I_b}{e} \ . \tag{6}$$

where $N_{ring} = I_b T_{rev} / e$ is the total number of electrons in the ring. Inserting eq. (2) and $I_b = 200mA$ into eq. (6) give

$$P_{beam} \approx 1.58MW \ . \tag{7}$$

2.




Since the two intersection points are on the light-cone opened-up by $x = (x_0, \vec{x})$, they satisfy the following equation:

$$(x_0 - X_0) - \sqrt{(X_1 - x_1)^2 + (X_2 - x_3)^2 + (X_3 - x_3)^2} = 0 \ , \tag{8}$$

and

$$(x_0 - Y_0) - \sqrt{(Y_1 - x_1)^2 + (Y_2 - x_3)^2 + (Y_3 - x_3)^2} = 0 \ . \tag{9}$$

Subtracting eq. (9) with eq. (8) yields

$$Y_0 - X_0 = \sqrt{(X_1 - x_1)^2 + (X_2 - x_2)^2 + (X_3 - x_3)^2} - \sqrt{(Y_1 - x_1)^2 + (Y_2 - x_2)^2 + (Y_3 - x_3)^2} \ . \tag{10}$$

The three points $\vec{x}$ , $\vec{X}$ and $\vec{Y}$ form a triangle and since the difference in the length of any two sides of a triangle is always smaller than the length of the third side, it follows from eq. (10)

$$Y_0 - X_0 \leq \sqrt{(X_1 - Y_1)^2 + (X_2 - Y_2)^2 + (X_3 - Y_3)^2} \ . \tag{11}$$

The time it takes for the particle to get from $\vec{X}$ to $\vec{Y}$ is given by

$$\Delta t = \frac{Y_0 - X_0}{c} \ , \tag{12}$$

and hence the average velocity of the particle during its travelling from $\vec{X}$ to $\vec{Y}$ is

$$\left\langle v_{particle} \right\rangle = \frac{\sqrt{\left(X_1 - Y_1\right)^2 + \left(X_2 - Y_2\right)^2 + \left(X_3 - Y_3\right)^2}}{\Delta t} = \frac{c\sqrt{\left(X_1 - Y_1\right)^2 + \left(X_2 - Y_2\right)^2 + \left(X_3 - Y_3\right)^2}}{Y_0 - X_0} \text{ .} \quad (13)$$

According to eq. (11) , the following relation holds

$$\frac{\sqrt{\left(X_1 - Y_1\right)^2 + \left(X_2 - Y_2\right)^2 + \left(X_3 - Y_3\right)^2}}{Y_0 - X_0} \geq 1 \text{ ,} \quad (14)$$

and inserting eq. (14) into eq. (13) yields

$$\left\langle v_{particle} \right\rangle = c\frac{\sqrt{\left(X_1 - Y_1\right)^2 + \left(X_2 - Y_2\right)^2 + \left(X_3 - Y_3\right)^2}}{Y_0 - X_0} \geq c \text{ .} \quad (15)$$

Eq. (15) violates special relativity and hence the trajectory of a particle cannot intersect a light-cone twice.

3. The angular distribution of radiation power is given by

$$\frac{dP(t_r)}{d\Omega}=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{4\pi c}\frac{\dot{\beta}^2}{(1-\beta\cos\theta)^3}\left[1-\frac{\sin^2\theta\cos^2\phi}{\gamma^2(1-\beta\cos\theta)^2}\right]. \qquad (1)$$

For $\frac{1}{\gamma^4}<<\theta<<1$ and $\gamma>>1$, we can use the following approximation

$$\begin{aligned}1-\beta\cos\theta &\approx 1-\beta\left(1-\frac{1}{2}\theta^2\right)\\ &=1-\beta+\frac{1}{2}\beta\theta^2\\ &=\frac{1}{\gamma^2(1+\beta)}+\frac{1}{2}\theta^2\\ &=\frac{1}{\gamma^2}\left[\frac{1}{2-(1-\beta)}\right]+\frac{1}{2}\theta^2 \quad , \qquad (2)\\ &\approx\frac{1}{2\gamma^2}\left[1+\frac{1-\beta}{2}\right]+\frac{1}{2}\theta^2\\ &\approx\frac{1}{2\gamma^2}\left[1+\frac{1}{4\gamma^2}+\ldots\right]+\frac{1}{2}\theta^2\\ &\approx\frac{1}{2\gamma^2}+\frac{1}{2}\theta^2\end{aligned}$$

and eq. (1) becomes

$$\frac{dP(t_r)}{d\Omega}\approx\frac{1}{4\pi\varepsilon_0}\frac{2e^2}{\pi c}\frac{\gamma^6\dot{\beta}^2}{(1+\gamma^2\theta^2)^3}\left[1-\frac{4\gamma^2\theta^2\cos^2\phi}{(1+\gamma^2\theta^2)^2}\right]. \qquad (3)$$

Since the factor inside the square bracket is between 0 and 1, the angular width of eq. (3) is determined by the factor $(1+\gamma^2\theta^2)^{-3}$, i.e. the radiation power drops substantially when $\theta\geq\frac{1}{\gamma}$ .

4. (a) The undulator period can be derived from the undulator equation with $\theta = 0$:

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$
$$\Rightarrow \lambda_u = \lambda \frac{2\gamma^2}{1+\frac{K^2}{2}} = 1.3cm \quad .$$

(b) The power radiated into the central cone is (slide #28, Lecture 16)

$$P_{cen} = \frac{\pi e \gamma^2 I_e}{\varepsilon_0 \lambda_u} \frac{K^2}{\left(1+\frac{K^2}{2}\right)^2} f(K) = 19.616W \ ,$$

where $f(K) = \left[ J_0\left(\frac{K^2}{4\left(1+\frac{K^2}{2}\right)}\right) - J_1\left(\frac{K^2}{4\left(1+\frac{K^2}{2}\right)}\right)\right]^2$. The photon flux is then

$$F_{cen} = \frac{P_{cen}}{\hbar\omega_0} = 3.95\times10^{16}s^{-1} \ ,$$

with $\omega_0 = 2\pi c/\lambda = 4.709\times10^{18} rad/s$. The spectral brightness is given by (slide #33 in Lecture 16)

$$B_{cen} = \frac{F_{cen}}{\Delta A \cdot \Delta\Omega \cdot N^{-1}} = \frac{F_{cen}}{2\pi\sigma_x\sigma_y\pi\theta_{Tx}\theta_{Ty}N^{-1}} = 1.668\times10^{35}m^{-2}s^{-1} = 1.668\times10^{20}\frac{1}{s\cdot mm^2 mrad^2\left(0.1\%BW\right)}$$

with $\theta_{Tx} = \sqrt{\theta_{cen}^2 + \sigma_{x'}^2} = 39.83\mu rad$ , $\theta_{Ty} = \sqrt{\theta_{cen}^2 + \sigma_{x'}^2} = 35.5\mu rad$ , $\theta_{cen} = \frac{1}{\gamma^*\sqrt{N}} = 35.4\mu rad$ , $\gamma^* = \frac{\gamma}{\sqrt{1+K^2/2}}$ , $\sigma_{x'} = \sqrt{\varepsilon_x/\beta_x} = 18.26\mu rad$ , $\sigma_{y'} = \sqrt{\varepsilon_y/\beta_y} = 2.58\mu rad$ , $\sigma_x = \sqrt{\varepsilon_x\beta_x} = 54.8\mu m$, and $\sigma_y = \sqrt{\varepsilon_y\beta_y} = 7.7\mu m$. One can also use the practical formula in slide #33 of Lecture 16 to get the answer.